King Fahd University of Petroleum and Minerals
College of Computing and Mathematics
Information and Computer Science Department
ICS 560: Fundamental of Quantum Computing
Semester 241
Quiz #1

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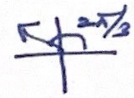
Name: …………………………………. ID #…………………………Date: 28/09/2024

Show all the necessary steps to earn full marks.

Q1: Let $z = 2e^{\frac{2\pi}{3}i}$. Write z in the standard form $z = a + bi$.

(2)

$z = 2(\cos \frac{2\pi}{3} + i \sin 2\pi/3)$



$= 2(-\frac{1}{2} + i\frac{\sqrt{3}}{2}) = -1 + i\sqrt{3}.$

Q2: Consider the following three vectors: $v_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $v_2 = \begin{bmatrix}2\\4\\6\end{bmatrix}$, and $v_3 = \begin{bmatrix}3\\6\\9\end{bmatrix}$.

Are these vectors linearly independent? Justify your answer with a detailed explanation.

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

(2)

$c_1 + 2c_2 + 3c_3 = 0 \rightarrow (1)$

$2c_1 + 4c_2 + 6c_3 = 0 \rightarrow (2)$

$3c_1 + 6c_3 + 9c_3 = 0 \rightarrow (3)$

From (1) & (2) $\Rightarrow$ $0 = 0 \Rightarrow$ infinitely solutions

Then $v_1, v_2,$ and v_3 are linear dependent

Q3: Given the vectors $v = \begin{bmatrix} 2+i \\ -i \\ 1-i \end{bmatrix}, u = \begin{bmatrix} 2-i \\ i \\ 1 \end{bmatrix}$, calculate the inner product of v and u.

$$\langle v, u \rangle = \begin{bmatrix} 2-i & i & 1+i \end{bmatrix} \begin{bmatrix} 2-i \\ i \\ 1 \end{bmatrix}$$

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$$= (2-i)^2 + i^2 + 1 + i$$
$$= 4 - 4i - 1 - \cancel{1} + \cancel{1} + i$$
$$= 3 - 3i$$

Q4: Determine whether the following vectors in are orthonormal? Justify your answer with a detailed explanation.

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$$v_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ i \end{bmatrix}, v_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 0 \\ i \end{bmatrix}, \text{ and } v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

orthogonal check

$$\langle v_1, v_2 \rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -i \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ i \end{bmatrix} = \frac{1}{2}(-1 + 0 + 1) = 0$$

$$\langle v_1, v_3 \rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$\langle v_2, v_3 \rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 0 & -i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0$$

Normalization check.

$$\langle v_1, v_1 \rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -i \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ i \end{bmatrix} = \frac{1}{2}(1 + 0 + 1) = 1$$

$$\langle v_2, v_2 \rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 0 & -i \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ i \end{bmatrix} = \frac{1}{2}(1 + 0 + 1) = 1$$

$$\langle v_3, v_3 \rangle = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 1$$

v_1, v_2, and v_3 are orthonormal

Q5: Determine if the following matrix is Hermitian or Unitary.

$$A = \begin{bmatrix} 2 & i \\ -i & 3 \end{bmatrix}$$

2

$A^\dagger = \begin{bmatrix} 2 & i \\ -i & 3 \end{bmatrix} = A$ Hermitian

$A^\dagger A = \begin{bmatrix} 2 & i \\ -i & 3 \end{bmatrix}\begin{bmatrix} 2 & i \\ -i & 3 \end{bmatrix} = \begin{bmatrix} 5 & 5i \\ -5i & 10 \end{bmatrix} = 5\begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix}$

$\neq I$

Not unitary

Q6: Let $v = \begin{bmatrix} 1+i \\ -1 \end{bmatrix}$ and $u = \begin{bmatrix} i \\ 3-2i \end{bmatrix}$. Compute the tensor Product $v \otimes u$ and $u \otimes v$.

2

$v \otimes u = \begin{bmatrix} (1+i)u \\ (-1)(u) \end{bmatrix} = \begin{bmatrix} -1+i \\ 5+i \\ -i \\ -3+2i \end{bmatrix}$

$u \otimes v = \begin{bmatrix} (i)v \\ (3-2i)v \end{bmatrix} = \begin{bmatrix} -1+i \\ -i \\ 5+i \\ -3+2i \end{bmatrix}$