King Fahd University of Petroleum and Minerals

College of Computing and Mathematics

Information and Computer Science Department

ICS 560: Foundation of Quantum Computing

Semester 251

Assignment 3

Show all necessary steps:

1. Compute $\langle 1+0|1-0\rangle$.

2. Consider the two-qubit state

$$\frac{1}{4}|00\rangle+\frac{1}{2}|01\rangle+\frac{1}{\sqrt{2}}|10\rangle+\frac{\sqrt{3}}{4}|11\rangle.$$

   If you measure only the left qubit, what are the resulting states, and with what probabilities?

3. Consider the three-qubit state

$$\frac{1}{6}|000\rangle+\frac{1}{3\sqrt{2}}|001\rangle+\frac{1}{\sqrt{6}}|010\rangle+\frac{1}{2}|011\rangle+\frac{1}{6}|100\rangle+\frac{1}{3}|101\rangle+\frac{1}{6}|110\rangle+\frac{1}{\sqrt{3}}|111\rangle.$$

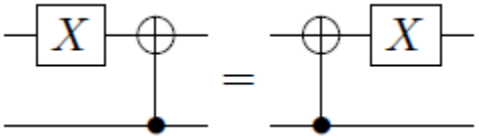
   If you measure only the left and right qubits, but not the middle qubit, what are the resulting states, and with what probabilities?

4. Are each of the following states seperable state or entangled state? If it is a product state, give the factorization.
   a) $\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle)$
   b) $\frac{1}{\sqrt{2}}(|10\rangle-i|11\rangle)$

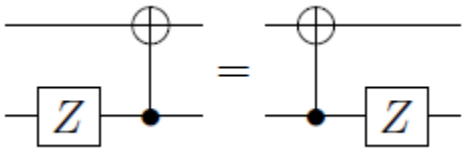
5. Are each of the following states a seperable state or entangled state? If it is a seperable state, give the factorization.
   a) $\frac{1}{4}\left(3|00\rangle-\sqrt{3}|01\rangle+\sqrt{3}|10\rangle-|11\rangle\right)$

b) $\frac{1}{\sqrt{3}}|0+\rangle + \sqrt{\frac{2}{3}}|1-\rangle$

6. Prove the following circuit identities, such as by finding the matrix representation of each circuit.
   a) $CNOT(I \otimes X) = (I \otimes X)CNOT$



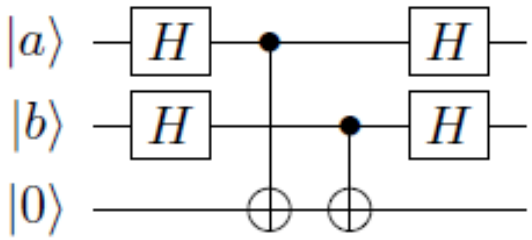
   b) $CNOT(Z \otimes I) = (Z \otimes I)CNOT$



7. The controlled-U (CU) gate applies some quantum gate U to the right qubit if the left qubit is 1. Write the controlled-Z and controlled-Y gates as a matrix

8. A unitary U that is able to clone qubits in two known state $|\psi\rangle$ That is, $U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$, Does there exist a quantum operator U that can clone both?
   a) $|+\rangle$ and $|i\rangle$
   b) $\frac{1+\sqrt{3}i}{4}|0\rangle + \frac{\sqrt{2}-i}{2}|1\rangle$ and $\frac{\sqrt{2}+i}{2}|0\rangle + \frac{-1+\sqrt{3}i}{4}|1\rangle$.

9. Consider the following quantum circuit:



a) If $|a\rangle = |-\rangle$ and $|b\rangle = |+\rangle$, find the resulting state at the end of the circuit.
b) If $|a\rangle = |-\rangle$ and $|b\rangle = |-\rangle$, find the resulting state at the end of the circuit.