King Fahd University of Petroleum and Minerals

College of Computing and Mathematics

Information and Computer Science Department

ICS 560: Fundamental of Quantum Computing

Semester 251

Assignment 1 (Complex Numbers and Complex Vector Space)

Show all your necessary steps:

1. Given the complex numbers $Z_1 = 3 + 4i$ and $Z_2 = 1 - 2i$, find the following

   a) $Z_1 + Z_2$ b) $Z_1 - Z_2$ c) $Z_1 \times Z_2$ d) $\frac{Z_2}{Z_1}$
2. Find the modulus of $Z = -5 + 12i$.
3. Convert the complex number $Z = -5 - 5i$ from algebraic to polar form (ρ, θ).
4. Write the complex number $Z = -4 + 4i\sqrt{3}$ in the Euler's form.
5. Find the cube roots of the complex number $Z = 125e^{0i}$.
6. Given the complex number $Z = 3e^{i\frac{\pi}{4}}$, find Z^4.
7. Rotate the complex number $Z = 5e^{30°i}$ by $90°$ counterclockwise. Express the resulting complex number in algebraic form.
8. Consider the vectors $v = \begin{bmatrix} 1+i \\ 2-i \\ 3 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ 1+i \\ 1-i \end{bmatrix}$, and $w = \begin{bmatrix} 3-i \\ i \\ 1 \end{bmatrix}$ in $\mathbb{C}^3$. Are these vectors linearly independent? Justify your answer.
9. Given the vectors $v = \begin{bmatrix} 1+i \\ 2 \\ 3-i \end{bmatrix}$, $u = \begin{bmatrix} 2-i \\ i \\ 1 \end{bmatrix}$, calculate the inner product $< u, v >$.
10. Find the norm (or length) of the vector $v = \begin{bmatrix} 2+i \\ -i \\ 1-i \end{bmatrix}$.
11. Given the matrix $A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$. Show that A is Hermitian matrix, Is this matrix A unitary? Justify your answer.

12. Show that the vectors $v = \begin{bmatrix} 1+i \\ 2-i \end{bmatrix}$ and $u = \begin{bmatrix} i \\ 1+2i \end{bmatrix}$ are linearly independent, then find the span of these vectors.

13. Given the following vectors in $\mathbb{C}^2$

$$v = \begin{bmatrix} \frac{3+i\sqrt{3}}{4} \\ \frac{1}{2} \end{bmatrix} \text{ and } u = \begin{bmatrix} \frac{1}{4} \\ x \end{bmatrix}.$$

Find the value of x such that v and u are orthogonal.

14. Consider the matrix $A = \begin{bmatrix} 1+i & 2 \\ -i & 1-i \end{bmatrix}$, find the eigenvalues of A.

15. Find the eigenvalues and its associated eigenvectors of the following matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$.

16. Show that A is Hermitian if and only if $A^T = \bar{A}$.

17. Given the Hermitian matrix $A = \begin{bmatrix} 3 & 2-i \\ 2+i & 1 \end{bmatrix}$, , find its eigenvalues. Verify that the eigenvalues are real.

18. Let $B = \begin{bmatrix} 2 & 1+i \\ 1-i & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Compute the matrix $D = B \cdot C$ and determine whether D is Hermitian.

19. Compute the tensor product $A \otimes B$, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

20. Compute the tensor product $A \otimes B$, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Determine whether $A \otimes B$ is Hermitian.