9.1 Quantum Oracle

A Boolean function is a function of the form:

$$f: \{0,1\}^n \to \{0,1\},$$

Boolean Function of **one bit**: $f: \{0,1\} \longrightarrow \{0,1\}$,

Example 1: Write the truth tale of $f\ (x) = \bar{x}$, where is $\bar{x}$ is NOT of x.

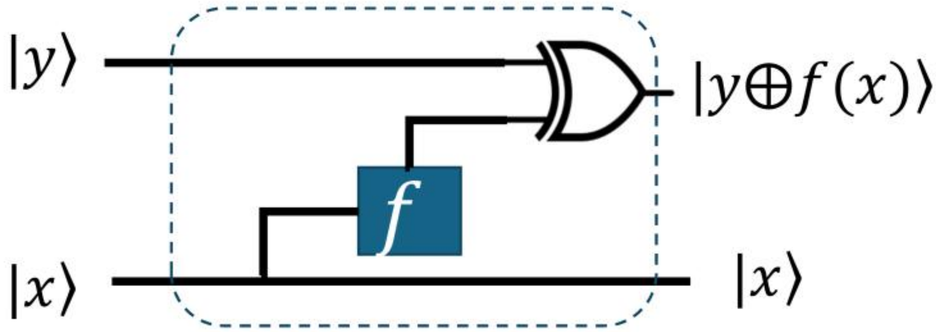
9.1 Quantum Oracle

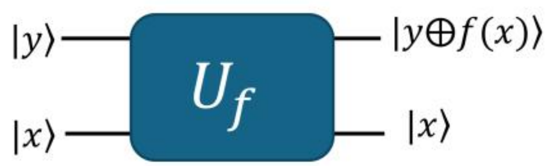
Example 2: Write the truth tale of $f\ (x, y)\ = x \wedge y$, where is $\bar{x}$ is NOT of x.

9.1 Quantum Oracle

Quantum oracle: is a black box.

Quantum gates are unitary operations which are invertible.





9.1 Quantum Oracle

Quantum Oracles for Functions of **Two Boolean** Inputs:

$$f: \{0,1\}^2 \longrightarrow \{0,1\}$$

$$|x_1, x_2, y\rangle \xrightarrow{U_f} |x_1, x_2, y \oplus f(x_1, x_2)\rangle$$

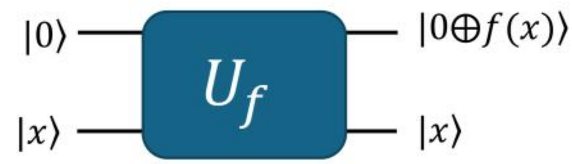
Quantum Oracles for Functions of **Boolean** Inputs:

$$f: \{0,1\}^n \longrightarrow \{0,1\}^m$$

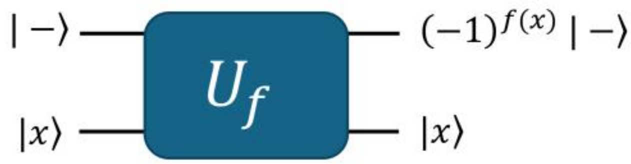
$$|x\rangle_n |y\rangle_m \xrightarrow{U_f} |x\rangle_n |y \oplus f(x)\rangle_m$$

9.1 Quantum Oracle

Quantum Oracle: for fix $y = 0$



Phase Oracle:



9.2 Deutsch's problem and algorithm

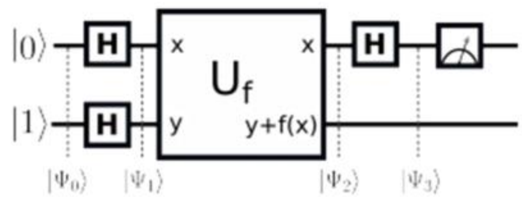
Consider a Boolean function $f: \{0,1\} \rightarrow \{0,1\}$,

How many possible values for $f(x)$ for all $x \in \{0,1\}$?

Classically: How many time to call the function to figure out if the function is constant or balanced?

9.2 Deutsch's problem and algorithm

Deutsch designs a quantum circuit that uses interference and a *single* function call to determine whether the function $f: \{0,1\} \rightarrow \{0,1\}$ is constant or balanced:



9.2 Deutsch's problem and algorithm

Example 3: Use **Deutsch's** algorithm to determine the function $f(x) = x$ is constant or balanced?

9.3 The N-bit Hadamard transformation

- The Hadamard operation applied to many or all qubits is a common element of many quantum algorithms.
- We consider a system of $N = 2^n$ qubits.
- The n −bit Hadamard transformation which is the tensor product of a Hadamard transformation for each bit.

9.3 The n −bit Hadamard transformation

Walsh-Hadamard transformation

$$W_n = H \oplus H \oplus \ldots . \oplus H = H^{\oplus n}$$

Example 4: Find W_3 on 3 qubits starting with $|\psi\rangle = |000\rangle$

9.3 The n −bit Hadamard transformation

Walsh-Hadamard transformation for n −qubit $|\psi\rangle = |00 \ldots 0\rangle$

$$W_n = H^{\oplus n}|\psi\rangle$$

Example 5: Find W_3 on 3 qubits starting with $|\psi\rangle = |001\rangle$

9.3 The n −bit Hadamard transformation

Example 6: Find W_3 on 3 qubits starting with $|\psi\rangle = |011\rangle$

We can define $x \cdot y$ for $x = (x_0, x_1, \dots, x_{n-1})$and $y = (y_0, y_1, \dots, y_{n-1})$are both $n - bit$ strings as follows

$$x \cdot y = x_0 \cdot y_0, + x_1 \cdot y_1 \dots + x_{n-1} \cdot y_{n-1} \; (mod\ 2)$$

$$W_n = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{x \cdot y} |x\rangle$$

9.4 Generalized Deutsch's Problem

Generalized Deutsch's Problem determine whether the function

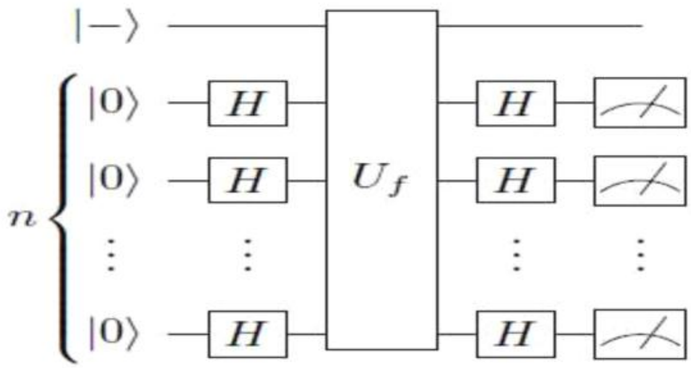
$f: \{0,1\}^n \rightarrow \{0,1\}$ is constant or balanced:

Classical Solution:

9.4 Generalized Deutsch's Problem

Quantum Solution: Deutsch-Jozsa Algorithm

Deutsch-Jozsa circuit



9.4 Generalized Deutsch's Problem

Example 7: Use Deutsch-Jozsa algorithm to show that $f(x_0, x_1) = 0$ is constant

9.4 Generalized Deutsch's Problem

Example 8: Use Deutsch-Jozsa algorithm to show that $f(x_0, x_1) = x_0 \oplus x_1$ is balanced