8.1 Superdense Coding Protocol

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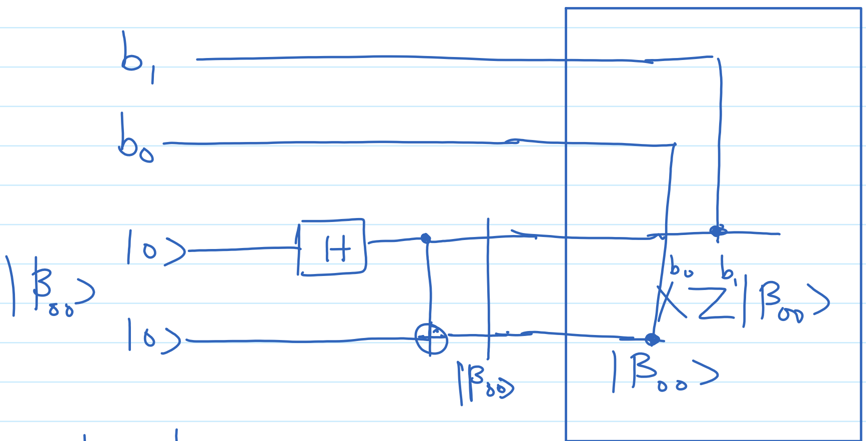
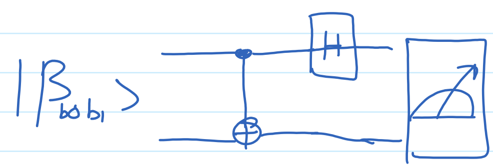
- Alice and Bob are living in separate places and Alice wants to send Bob some information.
- Alice would like to send Bob two classical bits of information over quantum communication channel.
- Alice and Bob sharing an entangled pair of particles.

$$\overset{|\beta_{00}\rangle}{\cancel{|\phi^+\rangle}} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Encoding

Decoding





$H|0\rangle = |+\rangle$

$H|+\rangle = H(H|0\rangle) = |0\rangle$

Decoding

$H(CNOT)|\beta_{00}\rangle$

$H\left(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\right)$

$= H\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)|0\rangle$

$= |0\rangle|0\rangle \Rightarrow (00)$

b_1 b_0

① 0 0 $Z^0X^0|\beta_{00}\rangle \xrightarrow{|\beta_{00}\rangle} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

② 0 1 $Z^0X^1|\beta_{00}\rangle \xrightarrow{\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)}$

① $CNOT\left(\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)\right)$

$= \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) = |+\rangle|1\rangle$

② $(H|+\rangle)|1\rangle = |01\rangle \xrightarrow{\text{measuring}} 01$

③ 1 0 $Z^1X^0|\beta_{00}\rangle \xrightarrow{\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)}$

① $CNot\left(\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)\right)$

$= \frac{1}{\sqrt{2}}\left(|00\rangle - |10\rangle\right) = |-\rangle|0\rangle$

$② (H|-\rangle)|0\rangle = |10\rangle$

$\longrightarrow 10$

④ 1 1 $\quad Z^1 X^1 |\beta_{00}\rangle \xrightarrow{\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)}$ $\quad$ ① $CNOT\left(\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)\right)$

$= \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle) = |-\rangle|1\rangle$

$② (H|-\rangle)|1\rangle = |11\rangle$

$\longrightarrow 11.$

Example 8.1.1: Consider the following state. Show that if Alice takes qubit 1 and Bob takes qubits 2 and 3, Alice can perform a local unitary operation and send Bob her qubit, which results in the transmission of two classical bits to Bob.

$$|\omega_1\rangle = \frac{1}{2}\left(|1\rangle_A|00\rangle_B + |0\rangle_A|10\rangle_B + \sqrt{2}|0\rangle_A|01\rangle_B\right)$$

Alice		
0	0	$I \otimes I \otimes I|\omega_1\rangle$
0	1	$X \otimes I \otimes I|\omega_1\rangle$
1	0	$Z \otimes I \otimes I|\omega_1\rangle$
1	1	$ZX \otimes I \otimes I|\omega_1\rangle$

8.2 Teleportation protocol

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- Alice wants to transmit an unknown quantum state to Bob

$$|\mathcal{X}\rangle = \alpha|0\rangle + \beta|1\rangle$$

Step 1: Alice and Bob share an entangled pair of Particles

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|0_A 0_B\rangle + |1_A 1_B\rangle\right)$$

Step 2: Alice combines her unknow qubit and applies a CNOT gate to her qubits

$$|\psi_0\rangle = |\mathcal{X}\rangle|\beta_{00}\rangle = \left(\alpha|0\rangle + \beta|1\rangle\right)\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|0 0_A 0\rangle + \alpha|0 1_A 1\rangle + \beta|1 0_A 0\rangle + \beta|1 1_A 1\rangle\right)$$

$$|\psi_1\rangle = CNOT \otimes I \; |\psi_0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$$

Step 3: Alice applies Hadamard gate to the first qubit of above result

$$|\psi_2\rangle = H \otimes I \otimes I \; |\psi_1\rangle$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|+\rangle|00\rangle + \alpha|+\rangle|11\rangle + \beta|-\rangle|10\rangle + \beta|-\rangle|01\rangle\right)$$

$$= \frac{1}{2}\left(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle\right)$$

$$= \frac{1}{2}\Big(|00\rangle\left(\alpha|0\rangle + \beta|1\rangle\right) + |01\rangle\left(\alpha|1\rangle + \beta|0\rangle\right) + |10\rangle\left(\alpha|0\rangle - \beta|1\rangle\right) + |11\rangle\left(\alpha|1\rangle - \beta|0\rangle\right)\Big)$$

Step 4: Alice measures her pair

ⓐ If Alice measure her qubits in $|00\rangle$

$$|\psi_2\rangle \xrightarrow{\text{Collapse}} \alpha|0\rangle + \beta|1\rangle = |\chi\rangle$$

ⓑ If measure in $|01\rangle \Longrightarrow \alpha|1\rangle + \beta|0\rangle$

$$\xrightarrow[\text{X gate}]{\text{apply}} \alpha|0\rangle + \beta|1\rangle = |\chi\rangle$$

ⓒ If she measure in $|10\rangle \Longrightarrow \alpha|0\rangle - \beta|1\rangle$

$$\xrightarrow[\text{Z gate}]{\text{Apply}} \alpha|0\rangle + \beta|1\rangle = |\chi\rangle$$

ⓓ If measure in $|11\rangle \Longrightarrow \alpha|1\rangle - \beta|0\rangle$

$$\xrightarrow[ZX]{\text{Apply}} \alpha|0\rangle + \beta|1\rangle$$

Step 5: Alice contact Bob over classical communication to tells him about her measurement

- Alice just calls Bob and for instances, says she got 01, then
- Bob applies his X gate to obtain the state Alice wanted to send to Bob.