# Review 1

Thursday, September 18, 2025 1:50 PM

Q6)

ⓐ $\langle v, u \rangle$

**1.** Given the complex numbers $Z_1 = 4 + 7i$ and $Z_2 = -2 + 3i$, find the following:

a) $Z_1 + Z_2$

b) $Z_1 - Z_2$

c) $Z_1 \times Z_2$

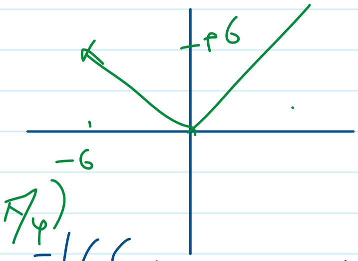
d) $\frac{Z_2}{Z_1}$

$-8$ $\quad 21$

$$\frac{Z_2}{Z_1} = \frac{-2+3i}{4+7i} \times \frac{4-7i}{4-7i} = \frac{13+26i}{16+49}$$

**2.** Find the modulus of $Z = 7 - 24i$.

$$|Z| = \sqrt{49+576} = \sqrt{\phantom{x}}$$



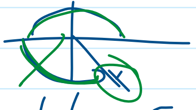
**3.** Convert the complex number $Z = -6 + 6i$ from algebraic to polar form $(\rho, \theta)$. $= (6\sqrt{2}, 3\pi/4)$

$$\theta = \tan^{-1}\left(\frac{6}{-6}\right) = \tan(-1)$$

$$= 3\frac{\pi}{4}$$

$$\rho = |Z| = \sqrt{36+36} = 6\sqrt{2}$$

$$Z = \rho e^{i\theta} = \rho(\cos\theta + i\sin\theta)$$



**4.** Write the complex number $Z = 2 - 2i\sqrt{3}$ in Euler's form.

$$\rho = |Z| = \sqrt{4+12} = 4, \quad \theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) = \tan^{-1}(-\sqrt{3})$$

$$= 5\frac{\pi}{3}$$

$$Z = 4e^{i5\pi/3}$$

$$\tan\theta = \sqrt{3}$$

$$\frac{\sin\theta}{\cos\theta} = \sqrt{3}$$

**5.** Find the fourth roots of the complex number $Z = 16e^{i\pi/2}$.

$$Z^{1/4} = 2e^{i\pi/8}$$

**8.** Consider the vectors

row → column

$(-1)^{i+j}$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$v = \begin{bmatrix} 2+i \\ -1+2i \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 1-i \\ 2 \\ i \end{bmatrix}, \quad w = \begin{bmatrix} i \\ 1+2i \\ -2 \end{bmatrix}$$

in $\mathbb{C}^3$. Are these vectors linearly independent? Justify.

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = 0$$

$3 \times 1 \quad 3 \times 3$

$$\det = 3\begin{vmatrix} 1-i & i \\ 2 & 1+2i \end{vmatrix} - i\begin{vmatrix} 2+i & 2 \\ -1+2i & 1+2i \end{vmatrix} + (-2)\begin{vmatrix} 2+i & 1-i \\ -1+2i & 2 \end{vmatrix}$$

$$\text{det} = 3\begin{vmatrix} 2 & 1+2i \end{vmatrix} - i\begin{vmatrix} -1+2i & 1+2i \end{vmatrix} + (-2)\begin{vmatrix} -1+2i & 2 \end{vmatrix}$$

$$= 3(3+i-2i)$$

$$\neq 0.$$

**9.** Given the vectors

$$v = \begin{bmatrix} 2-i \\ 3 \\ 1+i \end{bmatrix}, \quad u = \begin{bmatrix} i \\ 1-2i \\ 2 \end{bmatrix}$$

calculate the inner product $\langle u, v \rangle$.

$$\begin{bmatrix} -i & 1+2i & 2 \end{bmatrix}\begin{bmatrix} 2-i \\ 3 \\ 1+i \end{bmatrix} = -\cancel{2i} - 1 + 3 + 6i + 2 + \cancel{2i}$$

$$= 4 + 6i$$

**10.** Find the norm (or length) of the vector

$$(1-2i)(1+2i)$$

$$1^2 + 4$$

$$v = \begin{bmatrix} 1+2i \\ -3i \\ 2-i \end{bmatrix}.$$

$$|v| = \sqrt{\langle v, v \rangle} = \sqrt{1+4+9+4+1} = \sqrt{19}$$

**12.** Show that the vectors

$$\{ c v + c' u .$$

$$v = \begin{bmatrix} 1-i \\ 2+i \end{bmatrix}, \quad u = \begin{bmatrix} -i \\ 3 \end{bmatrix}$$

are linearly independent, then find the span of these vectors.

① & ②
② & ③
① & ③

$$c_1 v + c_2 u = 0$$

$$c_1(1-i) + c_2(-i) = 0 \Rightarrow \left( c_2 i = c_1(1-i) \right) \times -i$$

$$c_1(2+i) + 3c_2 = 0$$

$$c_2 = (-1-i)c_1$$

$$c_1(2+i) + c_1(-3-3i) = 0$$

$$c_1 \boxed{(-1-2i)} = 0 \qquad \boxed{c_1 = 0} \Rightarrow \boxed{c_2} = 0$$

**7.** Rotate the complex number $Z = 4e^{i45^\circ}$ by $120^\circ$ counterclockwise. Express the result in algebraic form.

$$Z = 4e^{i45^0} \cdot e^{i120}.$$

$$= 4e^{i165}$$

$$= 4(\cos(165^0) + i\sin(165^0)).$$

# Assignment 2(1)

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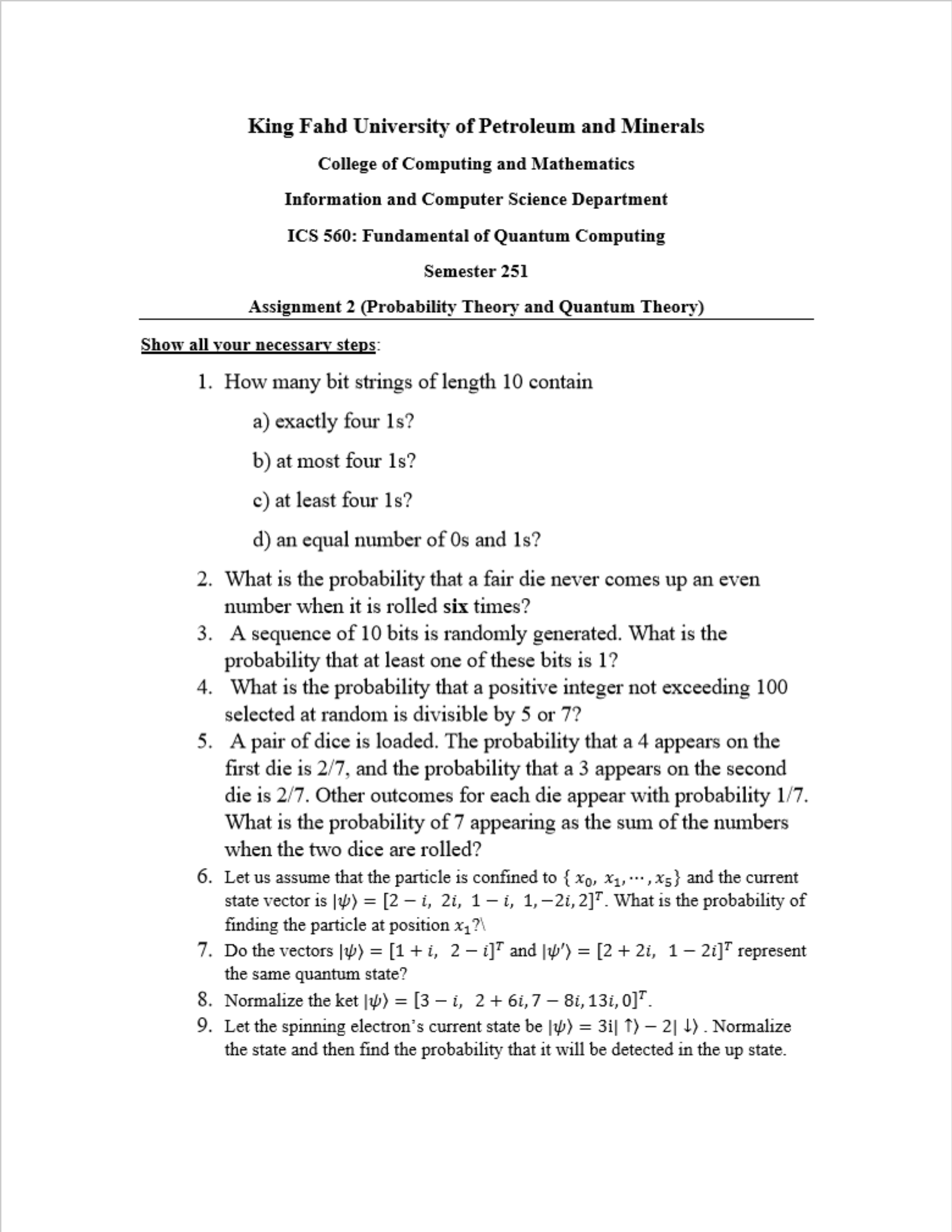
Assignment 2(1)



## King Fahd University of Petroleum and Minerals

**College of Computing and Mathematics**

**Information and Computer Science Department**

**ICS 560: Fundamental of Quantum Computing**

**Semester 251**

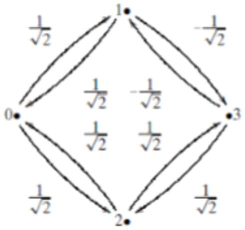
**Assignment 2 (Probability Theory and Quantum Theory)**

**Show all your necessary steps**:

1. How many bit strings of length 10 contain
   a) exactly four 1s?
   b) at most four 1s?
   c) at least four 1s?
   d) an equal number of 0s and 1s?
2. What is the probability that a fair die never comes up an even number when it is rolled **six** times?
3. A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is 1?
4. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7?
5. A pair of dice is loaded. The probability that a 4 appears on the first die is 2/7, and the probability that a 3 appears on the second die is 2/7. Other outcomes for each die appear with probability 1/7. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled?
6. Let us assume that the particle is confined to $\{ x_0,\ x_1, \cdots, x_5\}$ and the current state vector is $|\psi\rangle = [2-i,\ 2i,\ 1-i,\ 1, -2i, 2]^T$. What is the probability of finding the particle at position $x_1$?\
7. Do the vectors $|\psi\rangle = [1+i,\ \ 2-i]^T$ and $|\psi'\rangle = [2+2i,\ \ 1-2i]^T$ represent the same quantum state?
8. Normalize the ket $|\psi\rangle = [3-i,\ \ 2+6i, 7-8i, 13i, 0]^T$.
9. Let the spinning electron's current state be $|\psi\rangle = 3i|\uparrow\rangle - 2|\downarrow\rangle$ . Normalize the state and then find the probability that it will be detected in the up state.

10. Compute the amplitude of the transition from $|\psi\rangle = \frac{\sqrt{2}}{2}[i, -1]^T$ to $|\phi\rangle = \frac{\sqrt{2}}{2}[1, -i]^T$

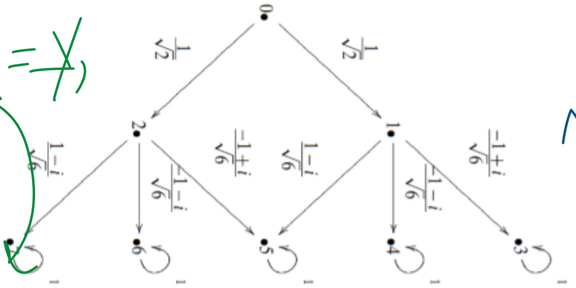
11. Consider the following graph



a) Construct the adjacency matrix of the above graph.

b) Is the resulting matrix unitary?

12. Consider the quantum system given by the following graph.



$MX = Y,$

$M^2 X$

$\|$

$MY$

$$
\begin{array}{c|cccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
0 & 0 & 0 & & & & & & \\
1 & 1/\sqrt{2} & 0 & & & & & & \\
2 & 1/\sqrt{2} & 0 & & & & & & \\
3 & 0 & \frac{1+i}{\sqrt{6}} & & & & & & \\
4 & 0 & \frac{-1-i}{\sqrt{6}} & & & & & & \\
5 & 0 & \frac{1-i}{\sqrt{6}} & & & & & & \\
6 & 0 & 0 & & & & & & \\
7 & 0 & 0 & & & & & &
\end{array}
$$

$M =$

a) Construct the adjacency matrix of the system.

b) If the particle at state $X = \left[\frac{1}{\sqrt{3}}, \sqrt{\frac{1}{5}}, 0, \frac{i}{\sqrt{15}}, 0, \frac{i}{\sqrt{15}}, \sqrt{\frac{2}{15}}, \sqrt{\frac{1}{5}}\right]^T$. Find the state of $X$ after one movement