King Fahd University of Petroleum and Minerals
College of Computing and Mathematics
Information and Computer Science Department
ICS 560: Foundations of Quantum Computing

Semester 251 Quiz #4

Name: ………………………………….. ID #.............................Date: 17/10/2025

Show all the necessary steps to earn full marks.

Q1 A particle is confined to the positions $\{x_0, x_1, x_2, x_3\}$ with state vector

$$|\psi\rangle = [1, i, -1, 2i]^T$$

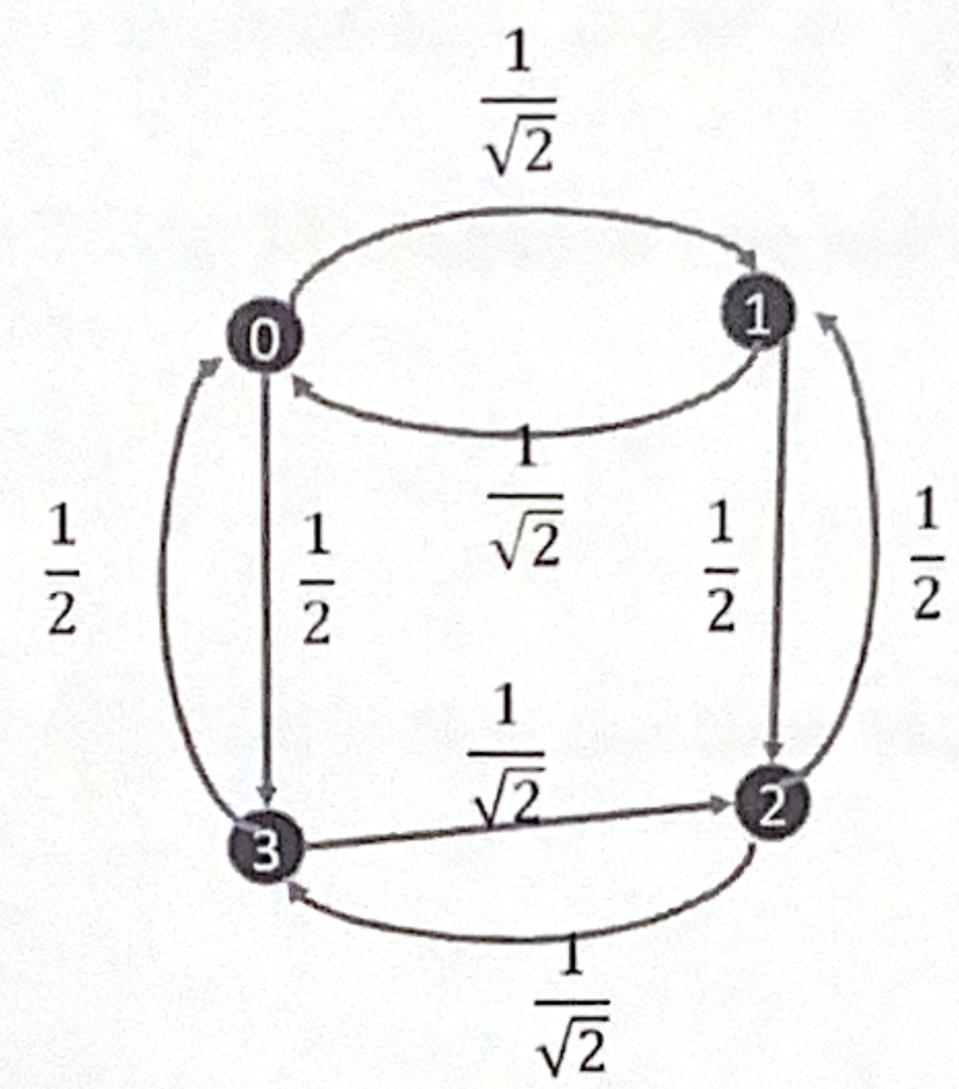
Find the **probability** of observing the particle at positions, x_0 and x_3.

$$\|\psi\|^2 = \sum_i |x_i|^2 = |1|^2 + |i|^2 + |-1|^2 + |2i|^2$$
$$= 1 + 1 + 1 + 4 = 7$$

$$p(x_0) = \frac{|x_0|^2}{7} = \frac{1}{7}$$

$$p(x_3) = \frac{|2i|^2}{7} = \frac{4}{7}$$

Q2: Consider the following **quantum system graph** with vertices $\{0,1,2,3\}$ and weighted edges:



a) Construct the **adjacency matrix** of the system.
b) Determine whether the matrix is **unitary**.

(a) $$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \end{array}$$

(b) $$A^{\dagger} A^{+} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{4} & 0 & \frac{1}{\sqrt{2}} & \\ & & & \\ & & & \end{bmatrix} \neq I$$

$\therefore$ A not unitary

Q3: Suppose a particle's current state is $|\psi\rangle = \frac{1}{2}[1, i. -i, -1]^T$.

If it evolves according to the **Hadamard operator**

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

acting on the qubit $(H \oplus H)|\psi\rangle$, find the **new state** after the transformation.

$$H \otimes H = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$(H \otimes H)|\psi\rangle = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ i \\ -i \\ -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0 \\ 2-2i \\ 2+2i \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 1-i \\ 1+i \\ 0 \end{bmatrix}$$

Q4: Multiple Choice Questions

1) Which of the following **best describes** a *doubly stochastic matrix*?

A. A matrix with complex entries whose row sums equal zero.

B. A real matrix whose row sums and column sums are all equal to 1

C. A matrix with real entries whose determinant equals 1.

D. A matrix with imaginary entries and nonzero trace.

2) Which statement best describes quantum interference?

A. Complex amplitudes can combine constructively or destructively, affecting probabilities.

B. Complex probability amplitudes always add to increase total probability.

C. Quantum amplitudes are always real and non-negative.

D. Interference occurs only in classical probability systems.

3) Which of the following statements correctly compares **classical probabilities** and **quantum probability amplitudes**?

A. Both are always real and non-negative.

B. Quantum amplitudes and classical probabilities are both complex quantities.

C. Classical probabilities can interfere destructively, unlike quantum amplitudes.

D. Quantum amplitudes can be complex, while classical probabilities are always real.