King Fahd University of Petroleum and Minerals
College of Computing and Mathematics
Information and Computer Science Department
ICS 560: Foundations of Quantum Computing

Semester 251 Quiz #2

Name: .....Key Answer............ ID #............................... Date: 19/09/2025

Show all the necessary steps to earn full marks.

1. Given $v = \begin{bmatrix} 2+i \\ -1 \\ 3-2i \end{bmatrix}$ and $u = \begin{bmatrix} 1-i \\ 4 \\ i \end{bmatrix}$, compute $\langle u, v \rangle$

$$\begin{bmatrix} 1+i & 4 & -i \end{bmatrix} \begin{bmatrix} 2+i \\ -1 \\ 3-2i \end{bmatrix}$$

$$= (1+i)(2+i) - 4 - i(3-2i)$$
$$= -5$$

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2. Find the distance between $v_1 = \begin{bmatrix} 1+2i \\ 3 \\ -i \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ -i \\ 4+i \end{bmatrix}$

$$d(v_1, v_2) = \|v_1 - v_2\| \qquad v_1 - v_2 = \begin{bmatrix} -1+2i \\ 3+i \\ -4-2i \end{bmatrix}$$

$$= \sqrt{\langle v_1 - v_2, v_1 - v_2 \rangle}$$

$$= \sqrt{1+4+9+1+16+4}$$

$$= \sqrt{35}$$

3

3. Determine if the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ i \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 4 \\ 2i \end{bmatrix}$ are linearly independent

$$c_1 v_1 + c_2 v_2 = 0$$

$-2\times \quad c_1 + 2c_2 = 0 \to (1) \quad \to 0 = 0$ (2)

$2c_1 + 4c_2 = 0 \to (2)$

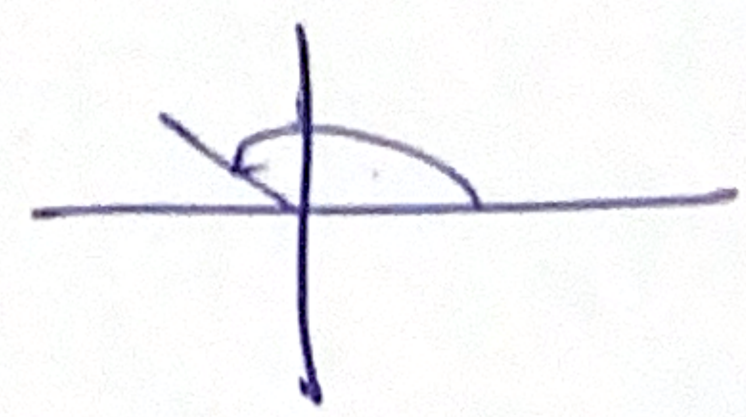
$c_1 i + 2i c_2 = 0$

$\therefore c_1 = -2c_2$

$\therefore$ v_1 and v_2 are linearly dependent

4. Find the cube roots of $\boldsymbol{z} = \mathbf{-8 + 8}\boldsymbol{i}$, and express the solutions both in polar and Cartesian forms.

$\rho = \sqrt{64+64} = 8\sqrt{2}$



$\theta = \tan^{-1}\left(\frac{8}{-8}\right) = \tan^{-1}(-1)$

$= 3\pi/4$

$\therefore Z = 8\sqrt{2}\, e^{i3\pi/4}$ (3)

$z^{1/3} = 2\sqrt[6]{2}\, e^{i\pi/4}$

$z^{1/3} = 2\sqrt[6]{2}\left(\cos \pi/4 + i \sin \pi/4\right)$

$= 2\sqrt[6]{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= \frac{2}{\sqrt[3]{2}}(1 + i)$

5. Let $v = \begin{bmatrix} 2+i \\ -1 \end{bmatrix}$ and $u = \begin{bmatrix} -i \\ x \end{bmatrix}$

Find the value of x such that v and u are orthogonal

$\langle v, u \rangle = 0$

$[2-i \quad -1] \begin{bmatrix} -i \\ x \end{bmatrix} = 0$

$-2i - 1 - x = 0$

$x = -1 - 2i$

2

6. **Multiple Choice Questions:**

1) For $v = \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$ and $u = \begin{bmatrix} i \\ 1 \end{bmatrix}$ the inner product $\cancel{\langle u, v \rangle}$ $\langle v, u \rangle$ =

A) $\mathbf{1+2i}$

B) $\mathbf{3+i}$

C) $\mathbf{2i}$

D) $\mathbf{-1+i}$

3

2) The norm of $v = \begin{bmatrix} 3-i \\ 2i \end{bmatrix}$ is

A) $\sqrt{13}$

B) $\sqrt{14}$

C) $\sqrt{15}$

D) $\sqrt{17}$

3) The cubic root of $z = 8e^{i\pi}$ $= -8$

A) 2

B) $2e^{i\pi/4}$

C) -2

D) 8