PHYS 512

Assignment – 03 $\langle Quantum | Computing \rangle$

Due Sunday Oct 04 – 11:59 pm (NO late submission)

Submit the solutions of Question 1, 4, 5, 7 and 9 (You are strongly encouraged to solve all of them)

Submit via email saleemg@kfupm.edu.sa. Clearly mark the subject 'PHYS 512- Assignment – 02"

Q.1 Ket and Bra, Orthonormality

Consider the following two kets:

$$|\psi\rangle = \begin{pmatrix} -3i \\ 2+i \\ 4 \end{pmatrix}, \qquad |\varphi\rangle = \begin{pmatrix} 2 \\ -i \\ 2-3i \end{pmatrix}$$

(i) Find the bra $\langle\varphi|$

(ii) Find $(|\psi\rangle)^*$

(iii) Are $|\psi\rangle$and $|\varphi\rangle$are orthogonal?

(iv) Briefly explain, why the product $|\psi\rangle|\varphi\rangle$ *and* $\langle\varphi|\langle\psi|$ do not make sense?

Q.2 Ket, Bra and inner product

Consider following quantum states

$$|\psi\rangle = 3i|\varphi_1\rangle - 7i|\varphi_2\rangle\,, \qquad |\chi\rangle = -|\varphi_1\rangle + 2i|\varphi_2\rangle$$

Where $|\varphi_1\rangle$ *and* $|\varphi_2\rangle$ are orthonormal. Calculate the inner (scaler) product $\langle\psi|\chi\rangle$ $\langle\chi|\psi\rangle$. Are they equal?

Q.3 Hermitian

Check the hermicity of operators $(\hat{A} + \hat{A}^\dagger)$, $i(\hat{A} + \hat{A}^\dagger)$, $i(\hat{A} - \hat{A}^\dagger)$,

Q.4 Outer product

Consider the two States

$$|\psi\rangle = i|\varphi_1\rangle + 3i|\varphi_2\rangle - |\varphi_3\rangle, \qquad |\chi\rangle = |\varphi_1\rangle - i|\varphi_2\rangle + 5i|\varphi_3\rangle$$

Where $|\varphi_1\rangle, |\varphi_2\rangle,$ *and* $|\varphi_3\rangle$ are orthogonal and normalized.

(i) Calculate$|\psi\rangle\langle\chi|$ *and* $|\chi\rangle\langle\psi|$. Are they equal?

(ii) Find Hermitian conjugate of $|\psi\rangle\langle\chi|$

Q.5 Hermitian

Consider the following operator:

$$\hat{A} = \begin{pmatrix} 0 & 0 & \frac{2}{1+i} \\ 0 & 0 & 0 \\ 1+i & 0 & 0 \end{pmatrix},$$

(v) Operator $\hat{A}$ is Hermitian?

(vi) Operator $\hat{A}$ is unitary?

(Clearly show all the working for Hermitian and unitary)

Q.6 Cauchy-Schwartz and Triangle inequality

Consider the following two states:

$$|\psi\rangle = -3|0\rangle - 2i|1\rangle\,, \qquad |\phi\rangle = |0\rangle + 5|1\rangle$$

Show that these two states obeys

(i) Cauchy-Schwartz inequality: $|\langle\psi||\phi\rangle|^2 \leq \langle\psi|\psi\rangle\langle\phi||\phi\rangle$

(ii) Triangle inequality: $\sqrt{\langle(\psi+\phi)|\psi+\phi\rangle} \leq \sqrt{\langle\psi||\psi\rangle} + \sqrt{\langle\phi||\phi\rangle}$

Q.7 Find new states

Consider following three quantum states

$$|\psi_1\rangle = |0\rangle, \qquad |\psi_2\rangle = -\tfrac{1}{2}|0\rangle - \tfrac{\sqrt{3}}{2}|1\rangle\,, \qquad |\psi_3\rangle = -\tfrac{1}{2}|0\rangle + \tfrac{\sqrt{3}}{2}|1\rangle$$

Find new states $|\tilde{\psi}_1\rangle, |\tilde{\psi}_2\rangle, and\ |\tilde{\psi}_3\rangle$ that are normalized, superposition states of $|0\rangle\ and\ |1\rangle$ and orthogonal to $|\psi_1\rangle, |\psi_2\rangle, and\ |\psi_3\rangle$, respectively.

Q.8 Direct Product/ combining quantum states

Show that the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\,,$$

Can be expressed in-terms of $|++\rangle\ and\ |--\rangle$ basis, i.e.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle\,,$$

Q.9 Expectation value

Consider the a states which is given in terms of three orthonormal vectors $|\varphi_1\rangle, |\varphi_2\rangle, and\ |\varphi_3\rangle$ as follows

$$|\psi\rangle = \frac{1}{\sqrt{15}}|\varphi_1\rangle + \frac{1}{\sqrt{3}}|\varphi_2\rangle + \frac{1}{\sqrt{5}}|\varphi_3\rangle$$

Where $|\varphi_n\rangle$ are eigenstates to an operator $\hat{B}$ such that $\hat{B}|\varphi_n\rangle = (3n^2 - 1)|\varphi_n\rangle\ with\ n = 1,2,3$

(iii) Find the norm of the state $|\psi\rangle$.

(iv) Find the expectation value of $\hat{B}$ for the state $|\psi\rangle$.

(v) Find the expectation value of $\hat{B}^2$ for the state $|\psi\rangle$.

Q.10 Define the mixed expression

In the following expression, where $\hat{A}$ is an operator, specify the nature of each expression (i.e. specify whether it is an operator, a ket, or a bra); then find it Hermitian conjugate.

(i) $\langle\psi|\hat{A}|\psi\rangle\langle\psi|$

(ii) $\hat{A}|\psi\rangle\langle\varphi|$

(iii) $(|\varphi\rangle\langle\varphi|\hat{A}) - i\left(\hat{A}|\psi\rangle\langle\psi|\right)$

Q.11 Combining quantum states

Consider following two quantum states in computational basis

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big|0\rangle - \frac{i}{\sqrt{2}}\Big|1\rangle, \qquad |\varphi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

Compute the following

(i) $|\psi\rangle\otimes|\varphi\rangle$

(ii) $\langle\psi|\otimes\langle\varphi|$

(iii) $(\sigma_x\otimes I)(|\psi\rangle\otimes|\varphi\rangle)$

(iv) $(\sigma_x\otimes\sigma_x)(|\psi\rangle\otimes|\varphi\rangle)$