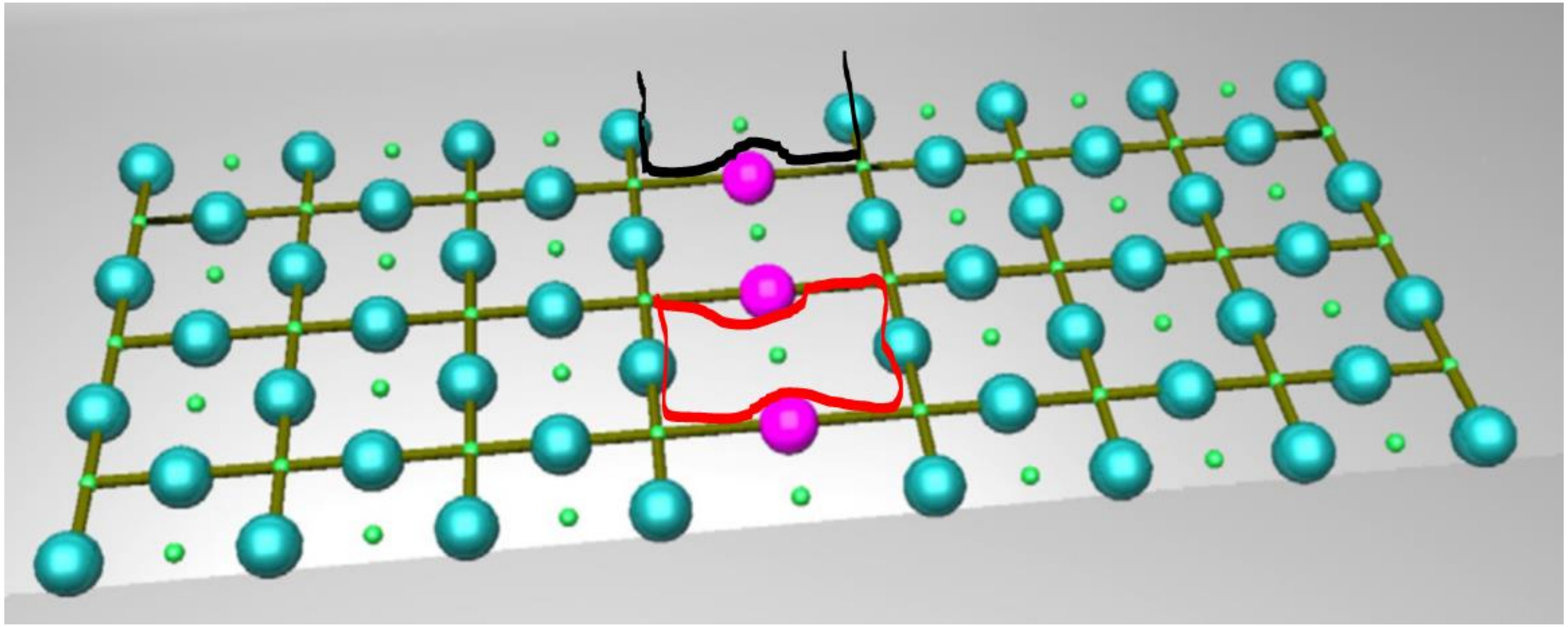
Quantum Error Correction – Surface Codes



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Outline

- From Exam Point of view focus on slides 10-25

References

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A. G. Fowler, D. S. Wang, Lloyd C. L. Hollenberg, **,**
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C. K. Andersen, A. Remm, S. Lazar, S. Krinner, N. Lacroix, G. J. Norris, M. Gabureac, C. Eichler, and A. Wallraff, ‘***Repeated Quantum Error Detection in a Surface Code***’, *Nat. Phys.* **16,** 875–880 (2020)

S. Krinner, …. A. Wallraff, ‘**Realizing Repeated Quantum Error Correction in a Distance-Three Surface Code**, Nature volume 605, pages669–674 (2022)

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Why we need a surface code

Error correction hardware can be not separated from computing hardware

Surface code is among the most promising platform
where we can integrate these requirements

Logical qubit can be modeled using 2D array of physical qubits

Surface Code

One approach to building a quantum computer is based on surface code

Realization of a 'surface code logical qubit' is key goal for many quantum computing hardware efforts

The challenge in creating quantum error correction codes lies in finding commuting sets of stabilizers that enable errors to be detected without disturbing the encoded information.

In terms of actual implementation, the specific **advantage of surface code for current hardware platforms** is that it requires only **nearest-neighbor interactions**. *(Short/long Interaction, with high fidelity, strongly depends upon hardware platform)*

One feature of the surface code is that **errors only need to be corrected when they effect measurement outcomes**

Surface Code Pros & Cons

For surface codes it is beneficial to adopt **a pictorial representation of the code qubits** in **place of the circuit notation**.

Price paid for the high error tolerance is that implementations of the **surface code involve large numbers of physical qubits**

It takes a minimum of **thirteen physical qubits** to implement a single logical qubit.

A **reasonably fault-tolerant logical qubit** that can be used effectively in a surface code takes of order **10^3 to 10^4 physical qubits**

The **number of physical qubits needed** to define a logical qubit is strongly dependent on the **error rate in the physical qubits**.

Surface Code

In the surface code, physical qubits are entangled together using a sequence of physical qubit CNOT operations, with subsequent measurements of the entangled states providing a means for error correction and error detection.

A set of physical qubits entangled in this way is used to define a logical qubit, which due to the entanglement and measurement has **far better performance than the underlying physical qubits**.

We will describe **how logical qubits are constructed in the surface code**, and also **show how the complete set of single logical qubit gates and the two-qubit logical CNOT are constructed**, allowing us to implement quantum algorithms based on these logical qubits.

Experimentalist view of QEC-Surface Code

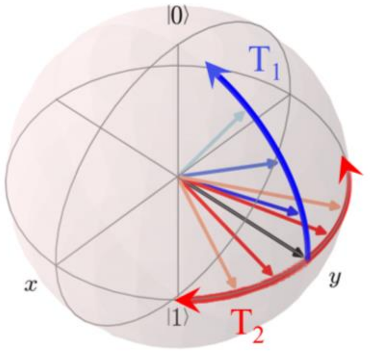
Introduction and key concepts related surface code

- 2D architect
- Parity measurements
- Understanding threshold
- Extension of logical states
- Some engineering related details of Surface code
- Constraints of system specification on error correction
- Experimental results surface code based QEC

Quantum Error Correction

The Challenge: Qubit Error

All physical qubit have two types of relaxation time, called T_1 and T_2

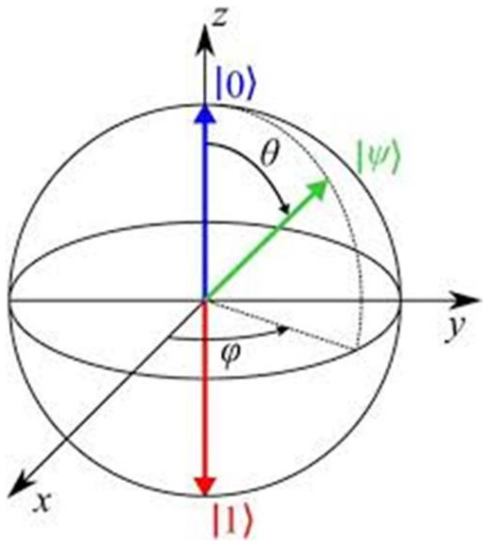


Control Errors

T_1 energy relaxation

T_2 Phase noise/evolution

Quantum gate error $\sim 10^{-2}\ -10^{-4}$ Required error rate $\sim 10^{-15} 10^{-20}$



$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi/2}\sin(\theta/2)|1\rangle$$

A quantum state has amplitude and phase

Qubit: Measure amplitude randomize the phase and vise versa

(Looks error correction is impossible!)

Qubits: Measure amplitude and phase parity at the same time

(Error correction is possible)

Amplitude and phase parity of two qubits can be measured at the same time

Parity

$00, 11 \rightarrow +$

$01, 10 \rightarrow -$

Error Detection in Surface Code (Commutation)

According to Heisenberg Uncertainty principle $[\hat{Z}, \hat{X}] = \hat{Z}\hat{X} - \hat{X}\hat{Z} \neq 0$

Bit flip error $\hat{X}|0\rangle = |1\rangle$ $\hat{X}|1\rangle = |0\rangle$

$$|0\rangle + |1\rangle \xrightarrow{\hat{X}} |1\rangle + |0\rangle \xrightarrow{\hat{Z}} -|1\rangle + |0\rangle$$

$$|0\rangle + |1\rangle \xrightarrow{\hat{Z}} |0\rangle - |1\rangle \xrightarrow{\hat{X}} |1\rangle - |0\rangle$$

$$\hat{Z}\hat{X} = -\hat{X}\hat{Z}$$

$$\hat{Z}\hat{X} + \hat{X}\hat{Z} = 0$$

$\hat{X}$ and $\hat{Z}$ anti commute

phase flip error $\hat{Z}|0\rangle = |0\rangle$ $\hat{Z}|1\rangle = -|1\rangle$

Simultaneous measurement is NOT possible

Parities commute

Simultaneous bit flip $\hat{X}_{12} = \hat{X}_1 \hat{X}_2$ Bit-flip on two qubits

Simultaneous phase flip $\hat{Z}_{12} = \hat{Z}_1 \hat{Z}_2$ Phase-flip on two qubits

$\hat{X}$ and $\hat{Z}$ operators commutes for two qubits

$$[\hat{X}_{12}, \hat{Z}_{12}] = \hat{X}_{12}\hat{Z}_{12} - \hat{Z}_{12}\hat{X}_{12} = \hat{X}_1 \hat{X}_2 \hat{Z}_1 \hat{Z}_2 - \hat{Z}_1 \hat{Z}_2 \hat{X}_1 \hat{X}_2$$

$$= \hat{X}_1 \hat{Z}_1 \hat{X}_2 \hat{Z}_2 - \hat{Z}_1 \hat{X}_1 \hat{Z}_2 \hat{X}_2$$

$$= \hat{X}_1 \hat{Z}_1 \hat{X}_2 \hat{Z}_2 - (-\hat{X}_1 \hat{Z}_1)(-\hat{X}_2 \hat{Z}_2)$$

$$= 0$$

Parities of pair behaves classically

Measuring $\hat{X}$ parity of one pair does not effect the $\hat{Z}$ parity of the pair

Stabilizers in Surface Code

The operator products $\hat{X}_1\hat{X}_2$ and $\hat{Z}_1\hat{Z}_2$ ($\hat{X}_1\hat{X}_2\ \hat{X}_3\hat{X}_4$, $\hat{Z}_1\hat{Z}_2\ \hat{Z}_3\hat{Z}_4$ etc.) **are stabilizers**.

Stabilizers are very important in preserving quantum states: by repeatedly measuring a quantum state using a set commuting stabilizers, the system is forced into a simultaneous and unique eigenstate of the all the stabilizers

One can measure the stabilizers without perturbing the system; **when measurement outcome change**, this corresponds to **one or more qubit error** and the quantum state is projected by the measurements onto a different stabilizer eigenstate

Error Detection

Parities behaviors classically (Simultaneous eigenstate)

Stable outcomes when both $\widehat{\boldsymbol{X}}_{\mathbf{12}}$ and $\widehat{\boldsymbol{Z}}_{\mathbf{12}}$ measured

($\hat{X}_{12}$measurements will NOT effect $\hat{Z}_{12}$ and vice versa)

$\hat{Z}_{12}$and $\hat{Z}_{23}$ parity measurements

Truth table for 3 qubits

1 2 3

$\hat{Z}_{12}$ $\hat{Z}_{23}$

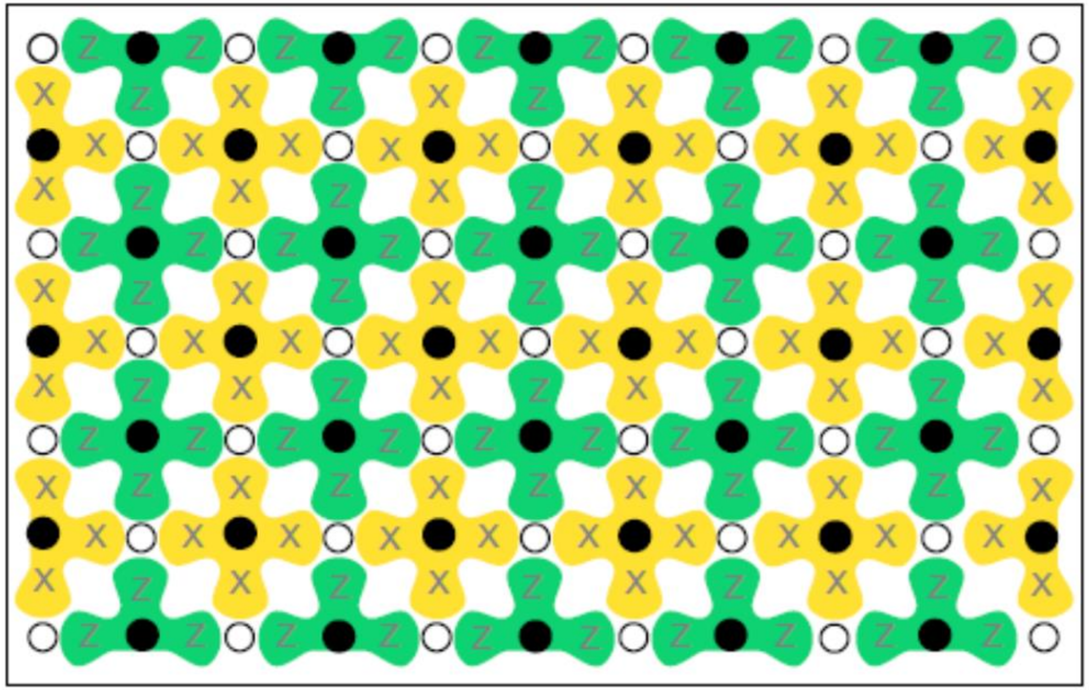
	Parities 12	Parities 23
000	**0**	**0**
001	**0**	**1**
010	**1**	**1**
011	**0**	**1**
100	**1**	**0**
101	**1**	**1**
110	**0**	**1**
111	**0**	**0**

Coding for single error

Assuming, only one bit flip

The Surface Code – 2D Array of Qubits

Surface code implement 2D array of physical qubits, the **qubits** are **either data** (open circles) in which the **computational quantum states are stored** and **measurement qubits** (filled circle)



All qubits meets basic requirements; ***initialization, single qubit rotation, and two-qubit CNOT between nearest neighbors***

Surface Code - Basic Concept

Consider a square lattice, put a qubit on each side of the square lattice

Define

Z-stabilizer for each square

$\hat{Z}_1\,\hat{Z}_3\,\hat{Z}_4\,\hat{Z}_6$

$\hat{Z}_7\,\hat{Z}_9\,\hat{Z}_{10}\,\hat{Z}_{12}$

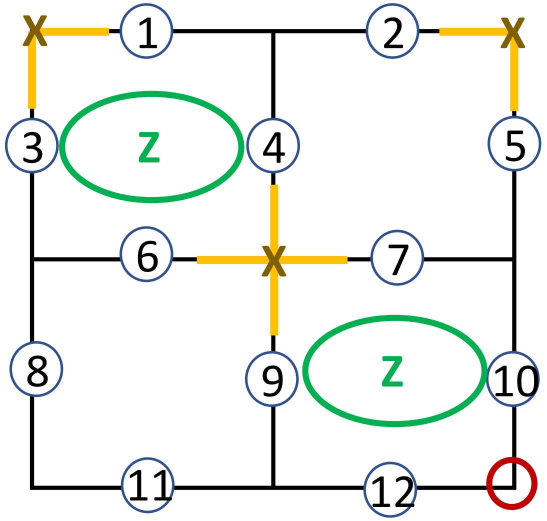
X-stabilizer for each vortex

$\hat{X}_4\,\hat{X}_6\,\hat{X}_7\,\hat{X}_9$

$\hat{X}_1\,\hat{X}_3$

$\hat{X}_2\,\hat{X}_5$

Stabilizers are adjacent to face or vortex



This grid can be very large with giant number of qubits

We can define 13 stabilizer for 12 qubits, where one is lineally independent, like

Array of qubits defines logical qubit

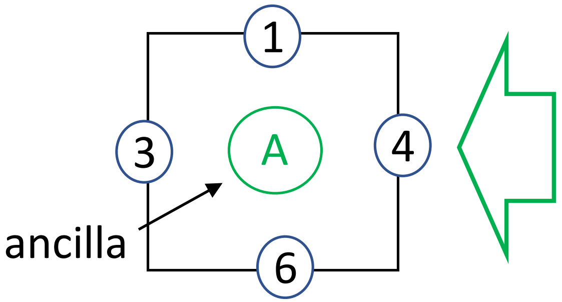
In Surface Code QEC we need to measure these stabilizers

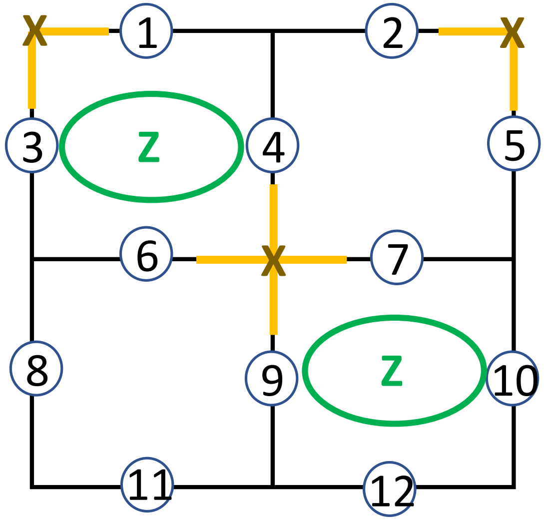
Measurement of Z-Stabilizers

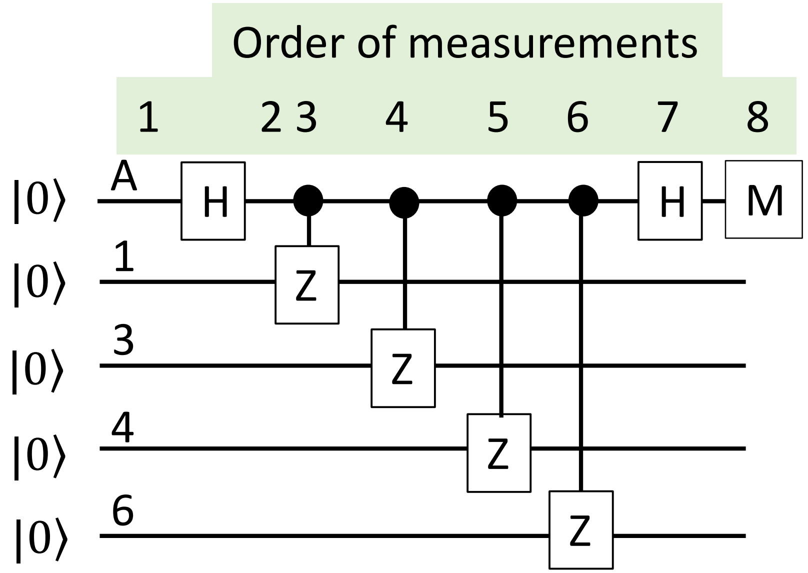
Let us measure one of the Z-stabilizer

(Z-syndrome measurements)

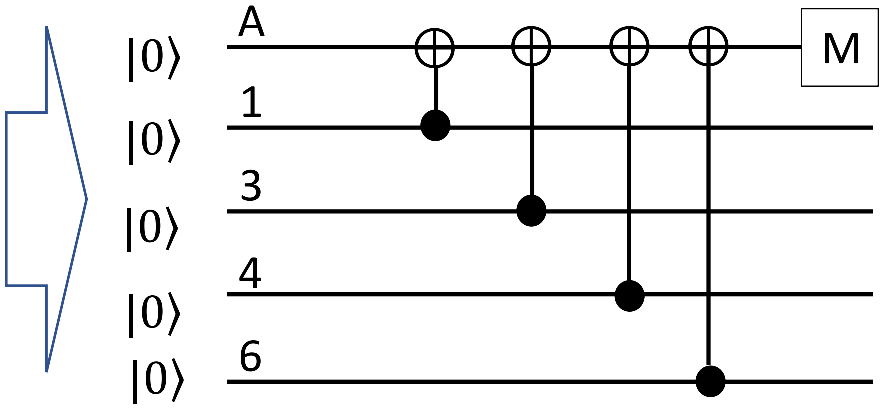
$$\hat{Z}_{1346} \equiv \hat{Z}_1 \, \hat{Z}_3 \, \hat{Z}_4 \, \hat{Z}_6$$







Measurement order is very important for stabilizers to commute



Eigenvalues of stabilizer $\hat{Z}_1 \, \hat{Z}_3 \, \hat{Z}_4 \, \hat{Z}_6$ will give +1 or -1

+1 mean no bit is flipped

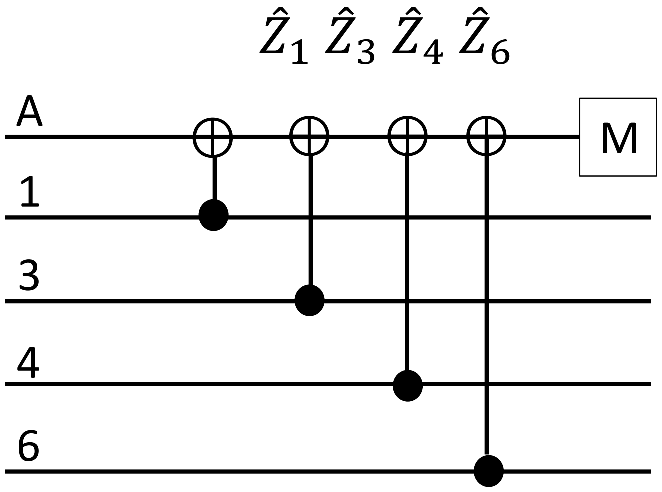
–1 mean a bit is flipped ⇨ Which one ? ⇨ **We don't know**

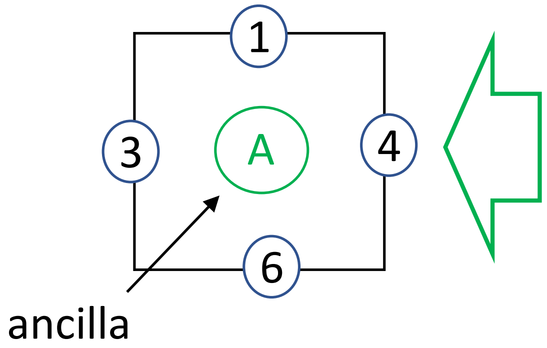
Similarly we can measure X-syndrome measurements

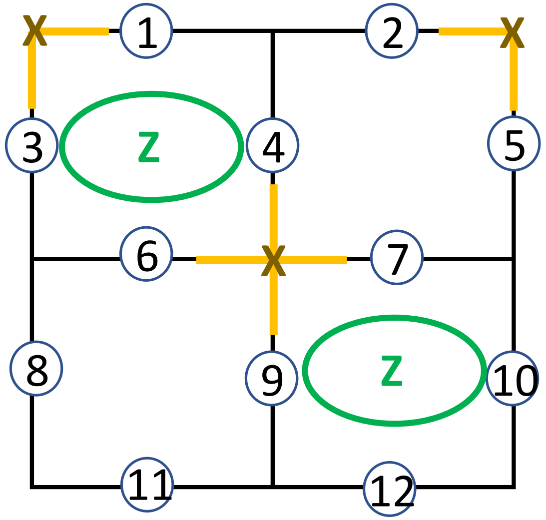
Measurement of Z-Stabilizers

Let us measure one of the Z-stabilizer

(Z-syndrome measurements)







Eigenvalues of stabilizer $\hat{Z}_1\ \hat{Z}_3\ \hat{Z}_4\ \hat{Z}_6$ will give +1 or -1

+1 mean no bit is flipped

–1 mean a bit is flipped ⇨ Which one ? ⇨ **We don't know**

Which of the bit is flipped?
(based upon these Z- results)

Z-measurements

+1	+1
+1	+1
-1	-1
+1	+1

Which of the bit is flipped?
(based upon these Z- results)

Z-measurements

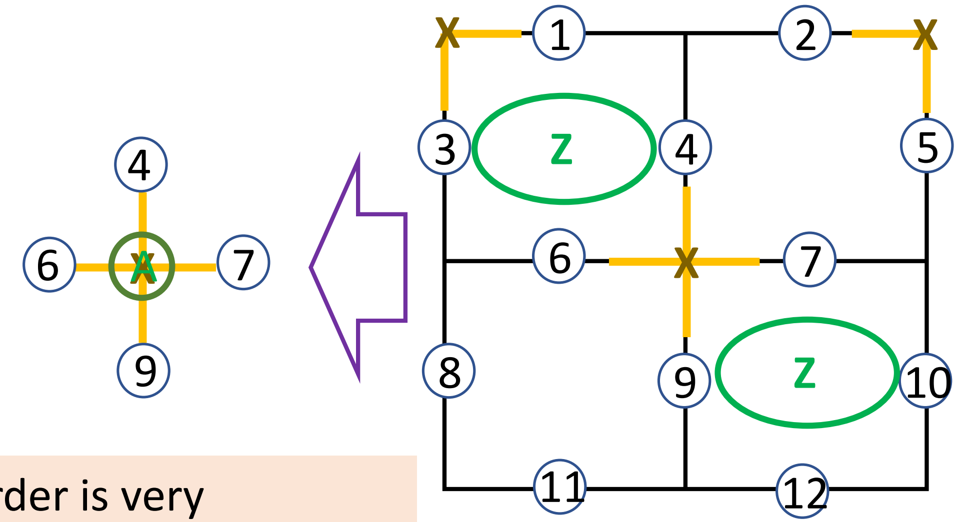
+1	+1
+1	+1
+1	+1
-1	+1

Measurement of X-Stabilizers

Let us measure one of the X-stabilizer

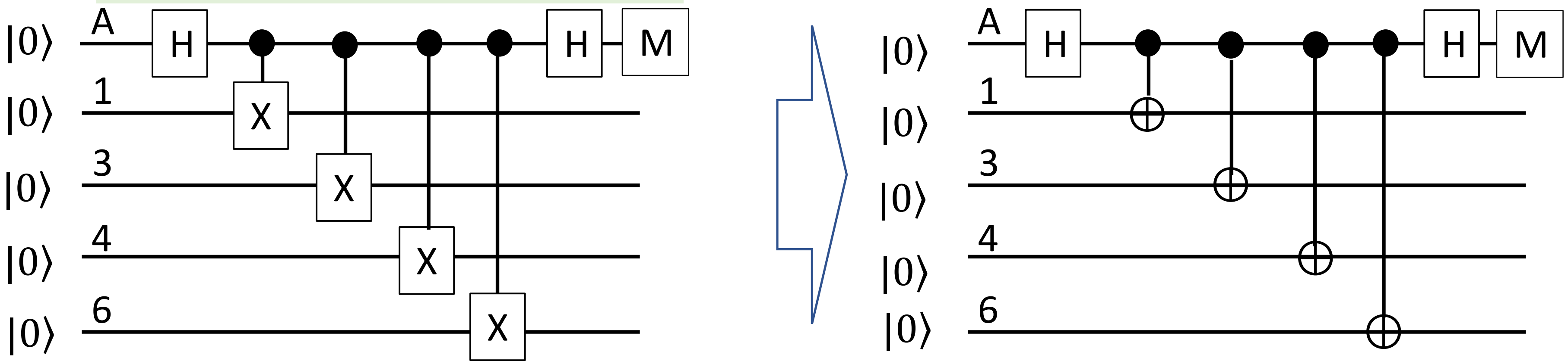
(X-syndrome measurements)

$$\hat{X}_4 \, \hat{X}_6 \, \hat{X}_7 \, \hat{X}_9$$



Order of measurements

1 2 3 4 5 6 7 8



Measurement order is very important for stabilizers to commute

Eigenvalues of stabilizer $\hat{X}_4 \, \hat{X}_6 \, \hat{X}_7 \, \hat{X}_9$ will give +1 or -1

+1 mean no bit is flipped

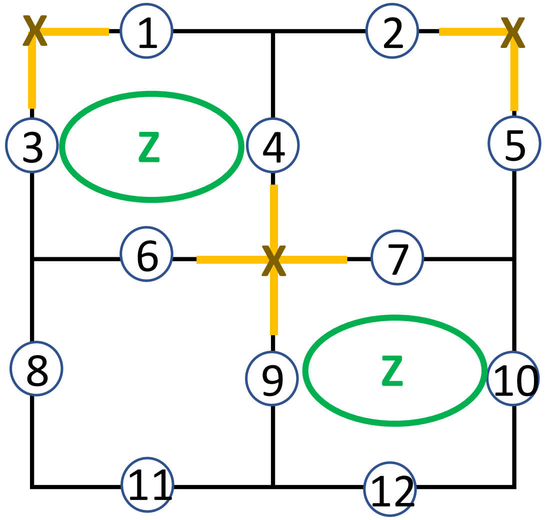
–1 mean a bit is flipped ⇨ Which one ? ⇨ **We don’t know**

Measurement of Stabilizers

Eigenvalues of stabilizer $\hat{X}_4\ \hat{X}_6\ \hat{X}_7\ \hat{X}_9$ will give +1 or -1

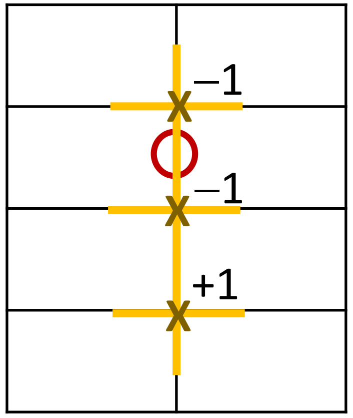
+1 mean no phase is flip error

–1 mean there is a phase flip error ⇨ Which one ? ⇨ **We don't know**



Where is phase flip error?
(based upon these Z- results)

X-measurements



X- and Z-stabilizer measurements can be used to find both the bit and phase flip error

Single Qubit Error

Erroneous single-qubit $\widehat{X}$ bit-flip **or $\widehat{Z}$** phase flip **operations**

These errors will be indicated by changes in the measurement outcomes

Consider a **single qubit error**, represented by the erroneous $\hat{I}_a + \epsilon \hat{Z}_a$ operating on data qubit a; $\epsilon \ll 1$ is a smaller number equal to the probability amplitude for the $\hat{Z}$ phase flip (neglect normalization)

The error transform the wave function $|\psi\rangle \rightarrow |\psi'\rangle = \left(\hat{I}_a + \epsilon \hat{Z}_a\right)|\psi\rangle$

Upon stabilizer (X and Z) **measurements**, the state $|\psi'\rangle$ project to state $|\psi\rangle$ with **near unit probability** and erases the error or projects it to $\hat{Z}_a\,|\psi\rangle$ with probability $|\epsilon|^2$, the later state is eigen state of all the stabilizers

$$\hat{X}_a\,\hat{X}_b\,\hat{X}_c\,\hat{X}_d\left(\hat{Z}_a\,|\psi\rangle\right) = -\hat{Z}_a\left(\hat{X}_a\,\hat{X}_b\,\hat{X}_c\,\hat{X}_d|\psi\rangle\right) = -\hat{Z}_a\,|\psi\rangle$$

Phase flip erroneous state is eigenstate of X-stab. with eigenvalue -1

$$\hat{Z}_a\,\hat{Z}_b\,\hat{Z}_c\,\hat{Z}_d\left(\hat{Z}_a\,|\psi\rangle\right) = -\hat{Z}_a\left(\hat{Z}_a\,\hat{Z}_b\,\hat{Z}_c\,\hat{Z}_d|\psi\rangle\right) = \hat{Z}_a\,|\psi\rangle$$

Phase flip erroneous state is eigenstate of Z-stab. with eigenvalue +1

Phase flip error can be fixed by applying $\hat{Z}_a$ again, however this phase flip cannot be applied with 100 % fidelity (could introduce more error in SC), instead it is safer to handle phase flip in software.

Surface Code-2D Array of Qubits

X syndrome measurements

Phase flip measure.

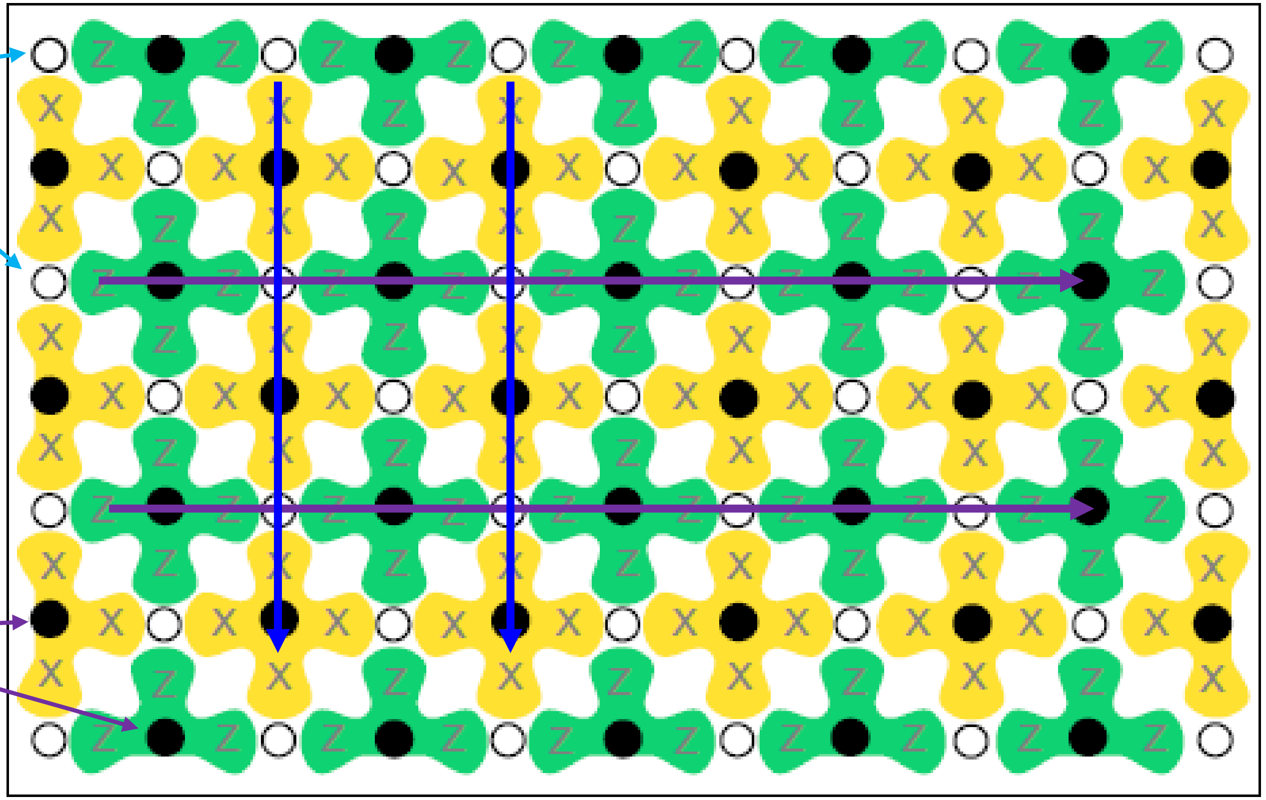
Logical Qubit

○ **Data qubit**

Z syndrome measurements

Bit flip measure.

● **Measurement** (ancilla) **qubit**

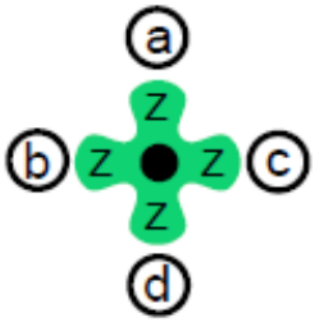


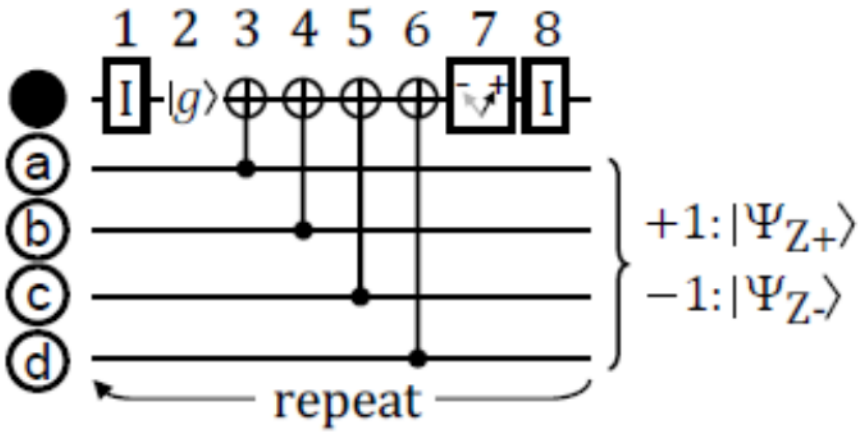
Small connectivity – **only among 4 nearest neighbor**, we can built on a chip

2D qubit structure we can built on chip

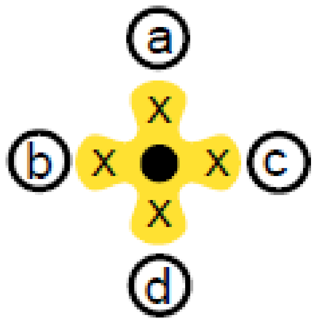
Surface Code

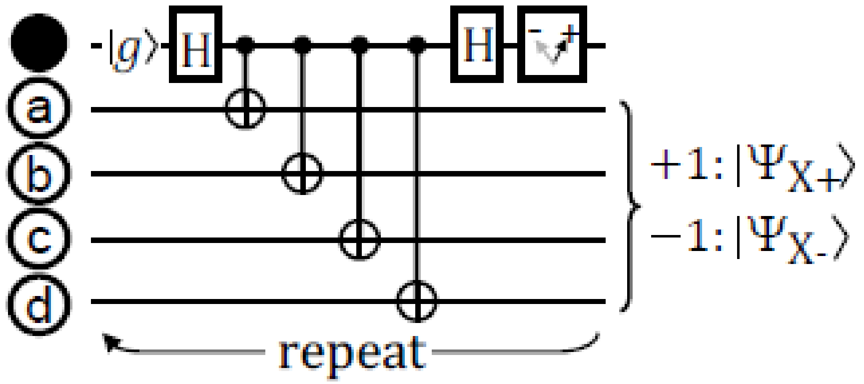
4 bit parity

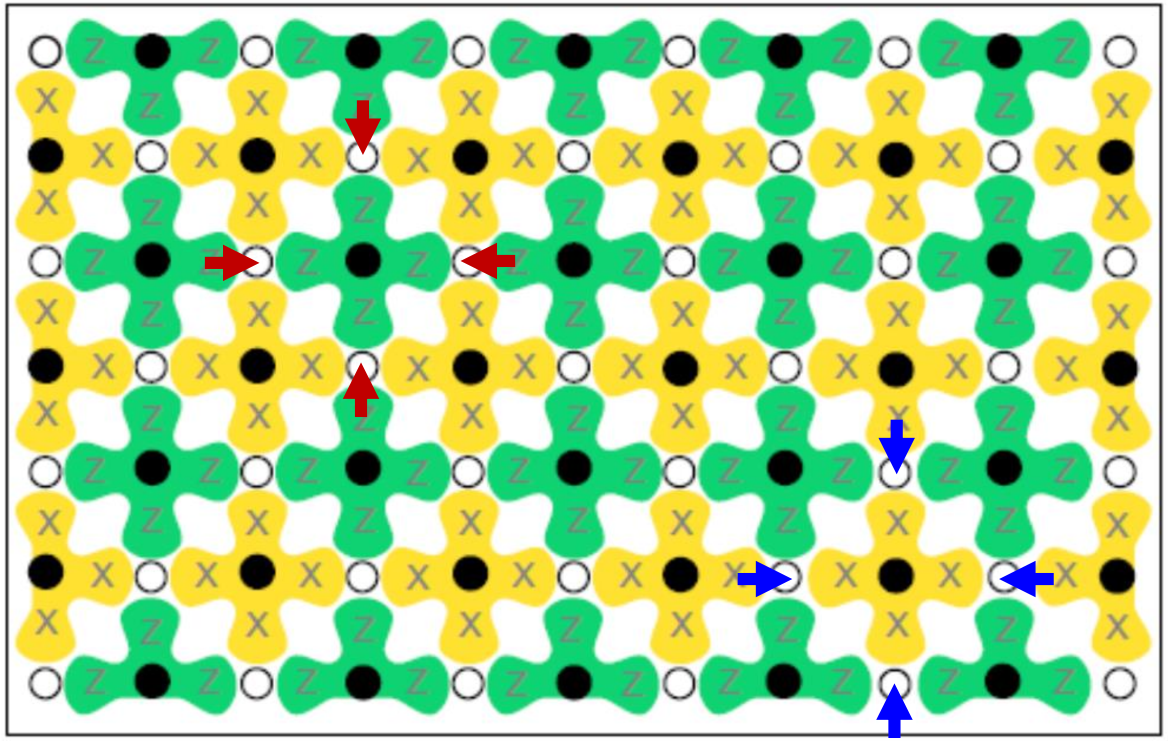




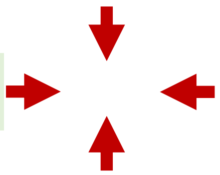
4 phase parity



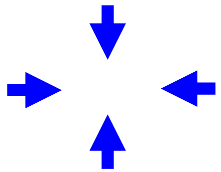




Parity (bit-flip) measurement 4 qubit



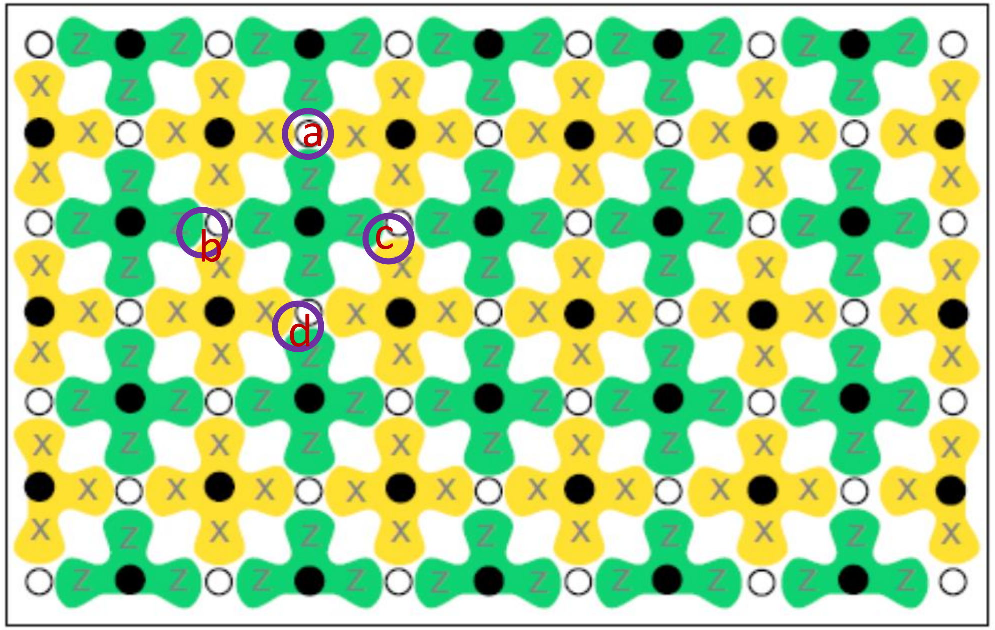
Parity (phase) measurement 4 qubit



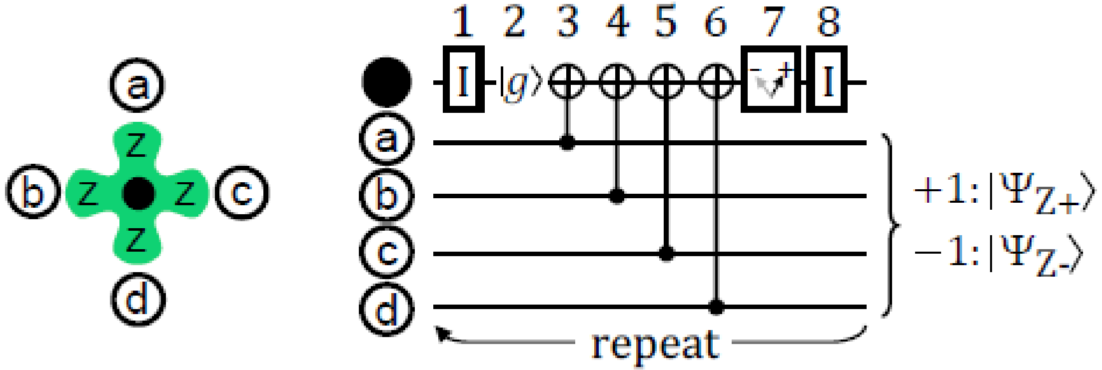
With multiple X- and Z- measurements exact error location can be determined

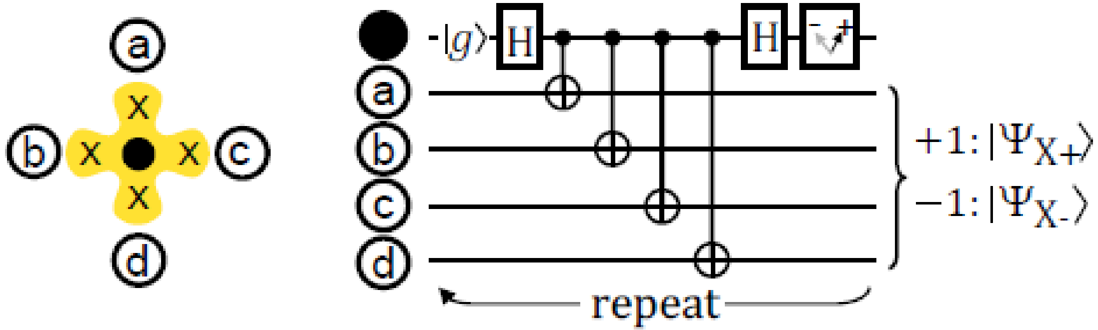
If any of the four bit/phase (a, b, c, d) is flipped (parity measurements) measurements out outcome is –1

Surface Code- Summary of Syndrome measurements



4 bit parity





4 phase parity

Eigenvalue	$\hat{Z}_a\hat{Z}_b\hat{Z}_c\hat{Z}_d$	$\hat{X}_a\hat{X}_b\hat{X}_c\hat{X}_d$
$+1$	$\|gggg\rangle$	$\|++++\rangle$
	$\|ggee\rangle$	$\|++--\rangle$
	$\|geeg\rangle$	$\|+--+\rangle$
	$\|eegg\rangle$	$\|--++\rangle$
	$\|egge\rangle$	$\|-++-\rangle$
	$\|gege\rangle$	$\|+-+-\rangle$
	$\|egeg\rangle$	$\|-+-+\rangle$
	$\|eeee\rangle$	$\|----\rangle$
-1	$\|ggge\rangle$	$\|+++-\rangle$
	$\|ggeg\rangle$	$\|++-+\rangle$
	$\|gegg\rangle$	$\|+-++\rangle$
	$\|eggg\rangle$	$\|-+++\rangle$
	$\|geee\rangle$	$\|+---\rangle$
	$\|egee\rangle$	$\|-+--\rangle$
	$\|eege\rangle$	$\|--+-\rangle$
	$\|eeeg\rangle$	$\|---+\rangle$

No bit/ phase-flip

Two bit/ phase-flip

One bit/ phase-flip

Three bit/ phase-flip

Eigenstates and eigenvalues for the 4-qubit stabilizers

Stabilized Sate and Identifying Qubit Error

All measurements XXXX and ZZZZ commutes

All parity measurements commutes

If there is bit flip error at ○

Which ancilla will detect it   E   E

If there is phase flip error at ○

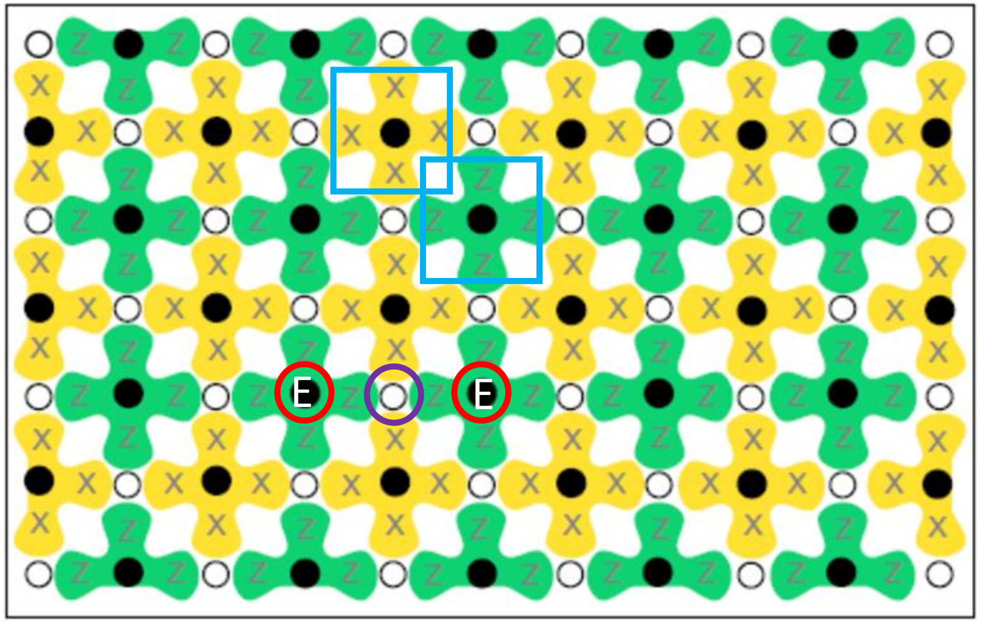
Which ancilla will detect?

Data qubits errors – pairs in space

Now we know the **error and its location**, apply suitable pulse and correct it (e.g. for bit flip, apply $\pi/2$ pulse)

$\hat{Z}_{12}$ and $\hat{X}_{12}$ commutes

(Z-measurements does not effect X measurements and vice versa)



39 data qubits, and 38 ancilla (measurement) qubits

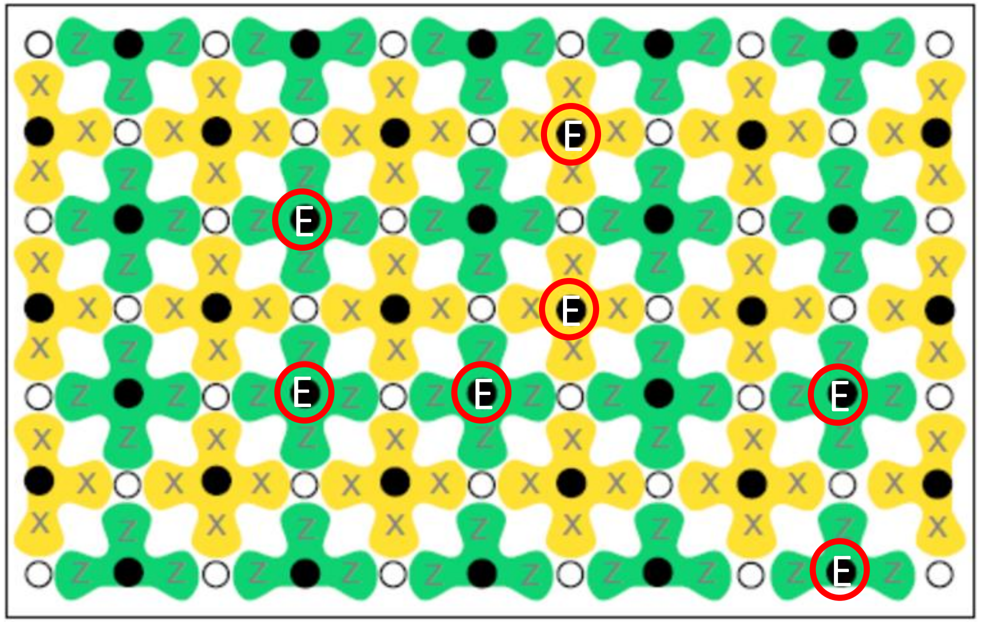
When measurements state collapse, still one data qubit left. That is the logical state (how quantum state encoded!)

Stabilized Sate and Identifying Qubit Error

Large density of errors!

How to fix them , may not be simple (no unique way)

Logical errors drops exponentially with bigger arrays (better than 99 % fidelity threshold)



If the errors are sufficiently rare, the error signals will be well-isolated on the 2D array, i.e. in space as well as in time. The error signals can then be matched up to deduce which specific qubit error occurred, with very high probability of correctly identifying the error

Syndrome Error Detection → Error in data Qubit

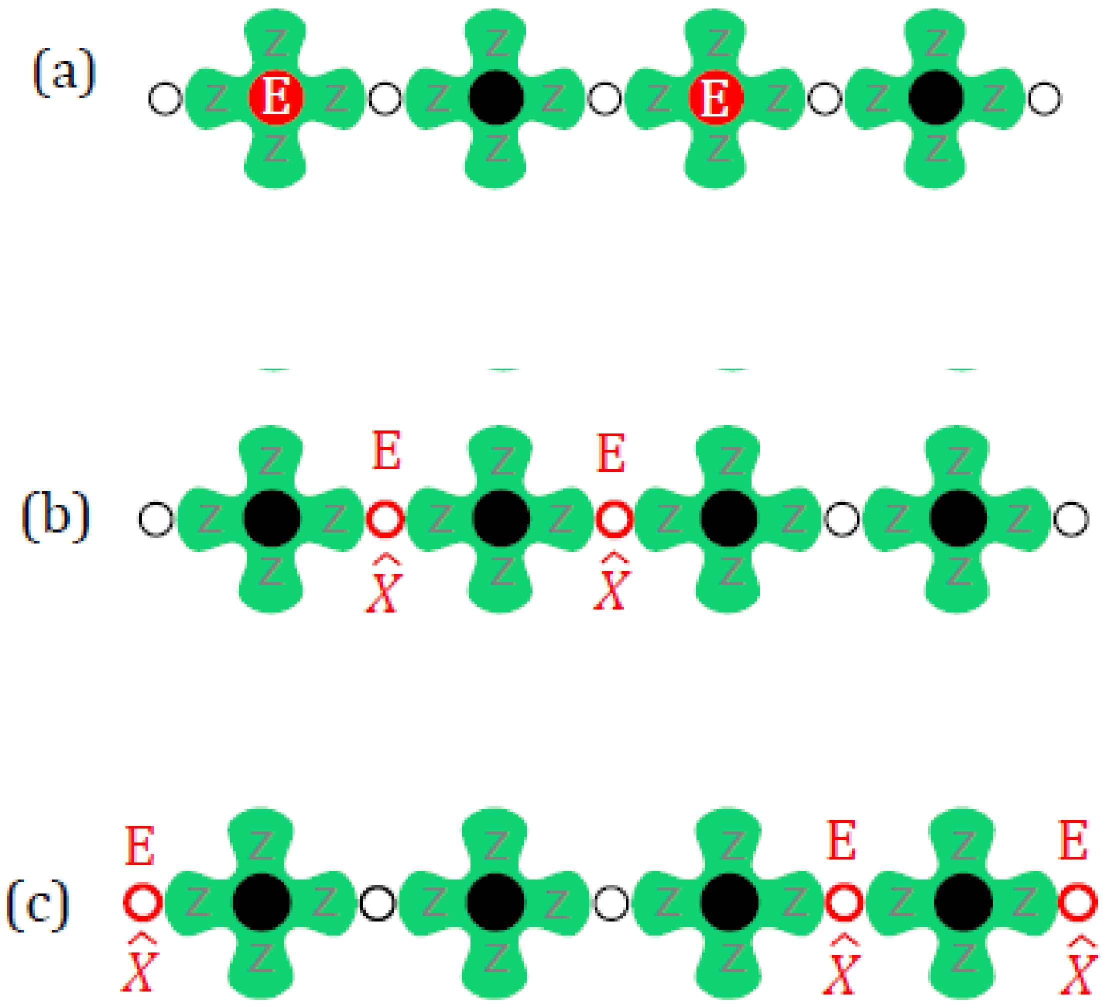
Error detected in measurement (ancilla) qubits (red)

Which data qubit(s) has an error ?

There are two possibilities Fig (b) and (c)

Probability of (b) is high – only ‘two errors’

Probability of (c) is low – ‘three errors’



Space and time Error Syndrome Measurements

Schematic evolution of measurement outcomes, over a segment of 2D array

Measurements in each horizontal plan, time progress upward

Errors occurring in measurement processes itself must also be considered

Such errors will yield in sign change for that measure qubit only and in next cycle will likely vanish (pair of sequential measurement is enough)

Other type of error such as CNOT errors, generate distinct patterns of sign changes in the measure-X and measure-Z qubits

Error rate depends upon the number of qubits, more the number of qubits, less the error is in logical qubit

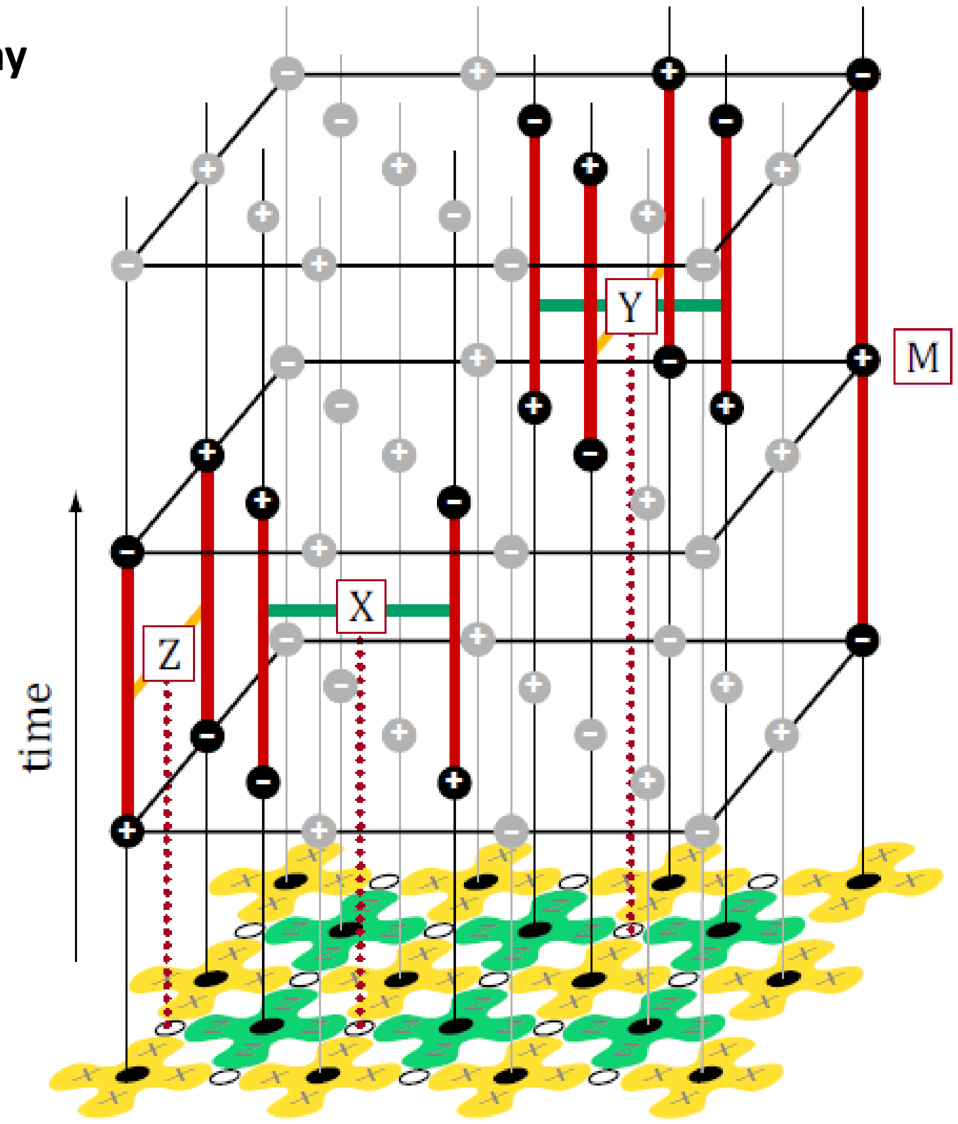


FIG. 2. (Color online) Schematic evolution of measurement outcomes (filled circles with ± signs), over a segment of the 2D array. Time progresses moving up from the array at the bottom of the figure, with measurement steps occurring in each horizontal plane. Vertical heavy red (gray) lines connect time steps in which a measurement outcome has changed, with the spatial correlation indicating an $\hat{X}$ bit-flip error, a $\hat{Z}$ phase-flip error, a $\hat{Y} = \hat{Z}\hat{X}$ error, and temporal correlation a measurement (M) error, which is sequential in time.

Logical Operator (Logical Qubit)
(we have 40 data and 39 ancilla)

$\hat{Z}_L = \hat{Z}_{12345} \equiv \hat{Z}_1\, \hat{Z}_2\, \hat{Z}_3\, \hat{Z}_4\, \hat{Z}_5$

$\hat{X}_L = \hat{X}_{678910} \equiv \hat{X}_6\, \hat{X}_7\, \hat{Z}_8\, \hat{Z}_9\, \hat{Z}_{10}$

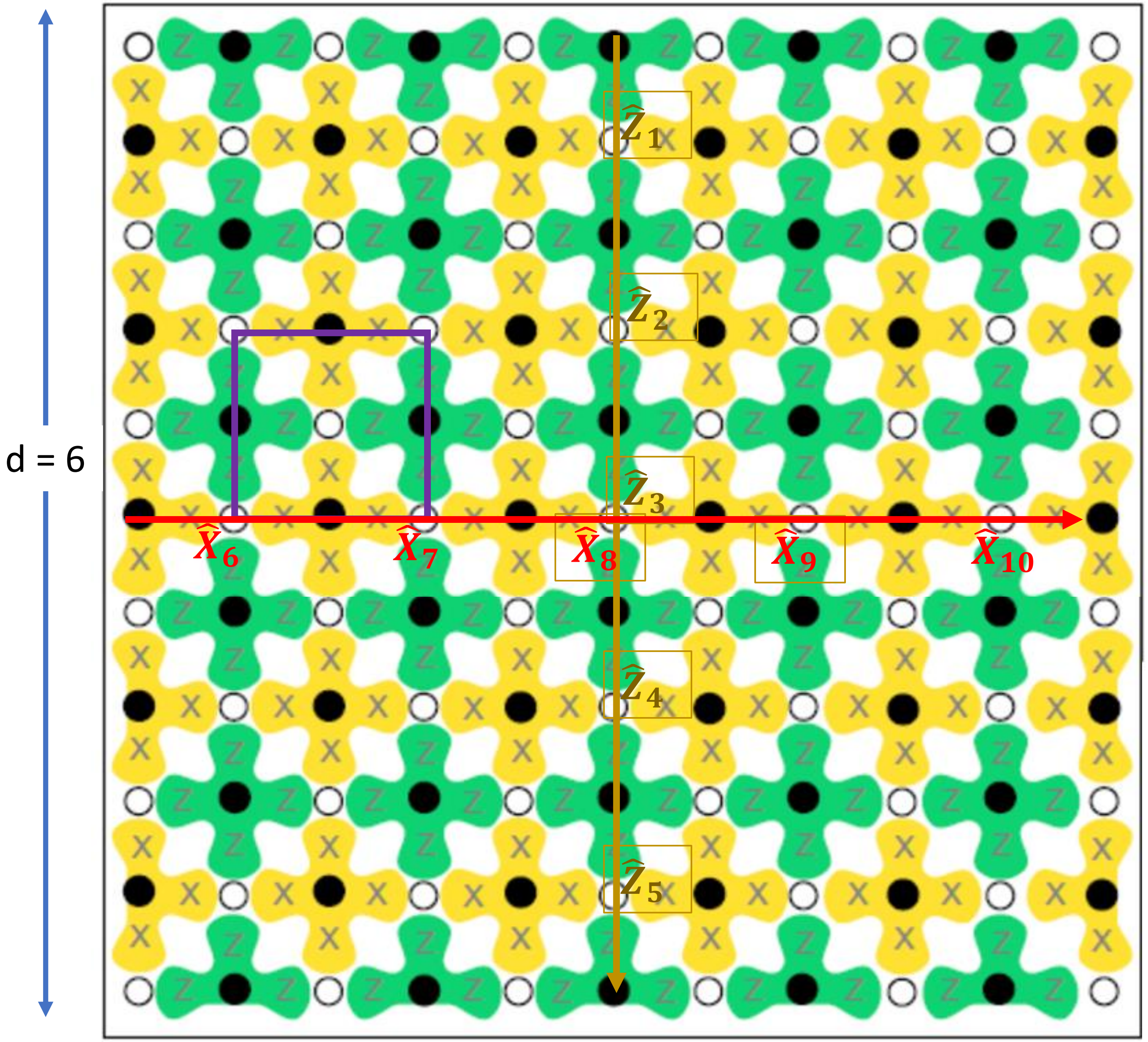
$\hat{X}_L\, \hat{Z}_L = -\hat{Z}_L\, \hat{X}_L$ Logical qubit anti-commute

$\hat{X}_8, \hat{Z}_3$ is the same and $\hat{X}\hat{Z}$ do not commute for a qubit

$\hat{X}_L$and $\hat{Z}_L$ belongs to quantum bit – logical qubit

Using $\hat{X}_L$and $\hat{Z}_L$ we can define a logical qubit and know their eigen values (± 1)

What is exactly 'the logical qubit'- we do not need it



Logical Qubit with Z-cut

Solid black line (top) array boundary, other three sides of array continue outwards indefinitely

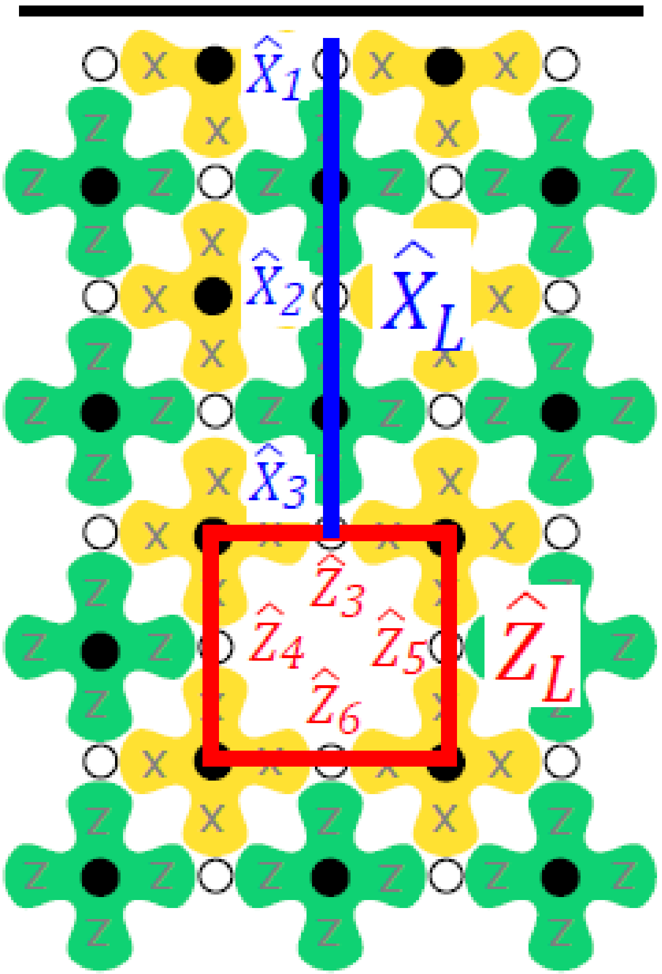
Two logical operator

$\hat{X}_L = \hat{X}_{123} \equiv \hat{X}_1\hat{X}_2\hat{X}_3$ **(Array from outer X-boundary)**

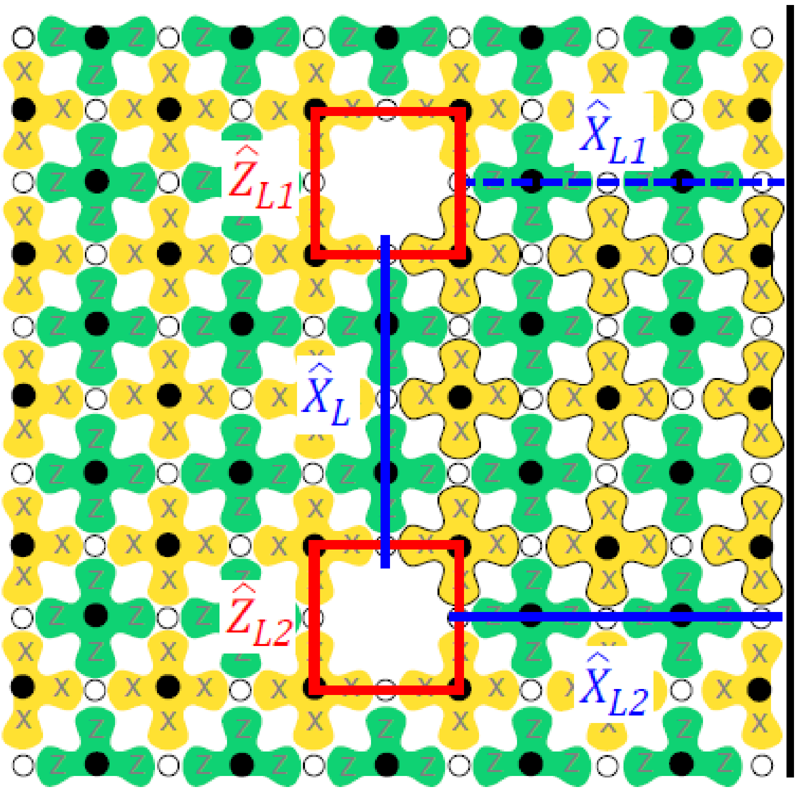
$\hat{Z}_L = \hat{Z}_{3456} \equiv \hat{Z}_3\hat{Z}_4\hat{Z}_5\hat{Z}_6$ **(Surrounding the Z-cut hole)**

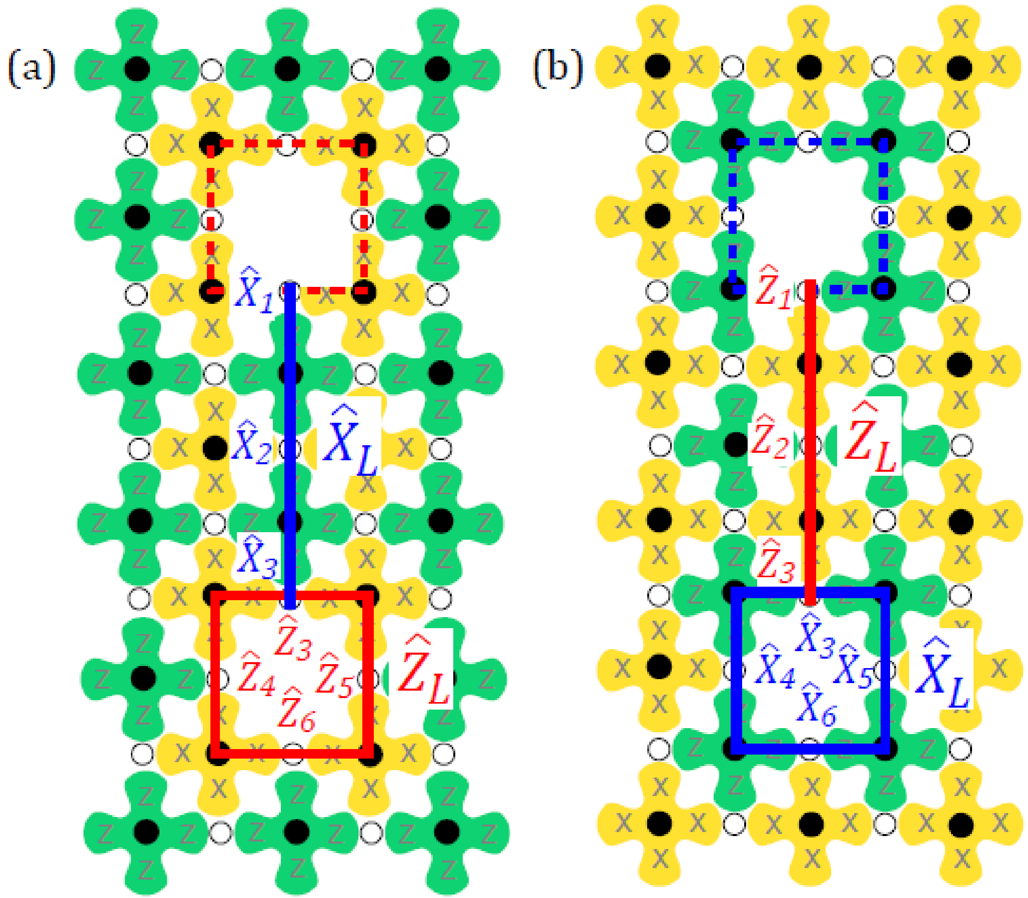
The operator chain for $\widehat{X}_L$ and $\widehat{Z}_L$ have one Physical data qubit (3) in common

$\hat{X}_L\,\hat{Z}_L = -\hat{Z}_L\,\hat{X}_L$ **Logical qubit anti-commute**



Logical Qubits - few more





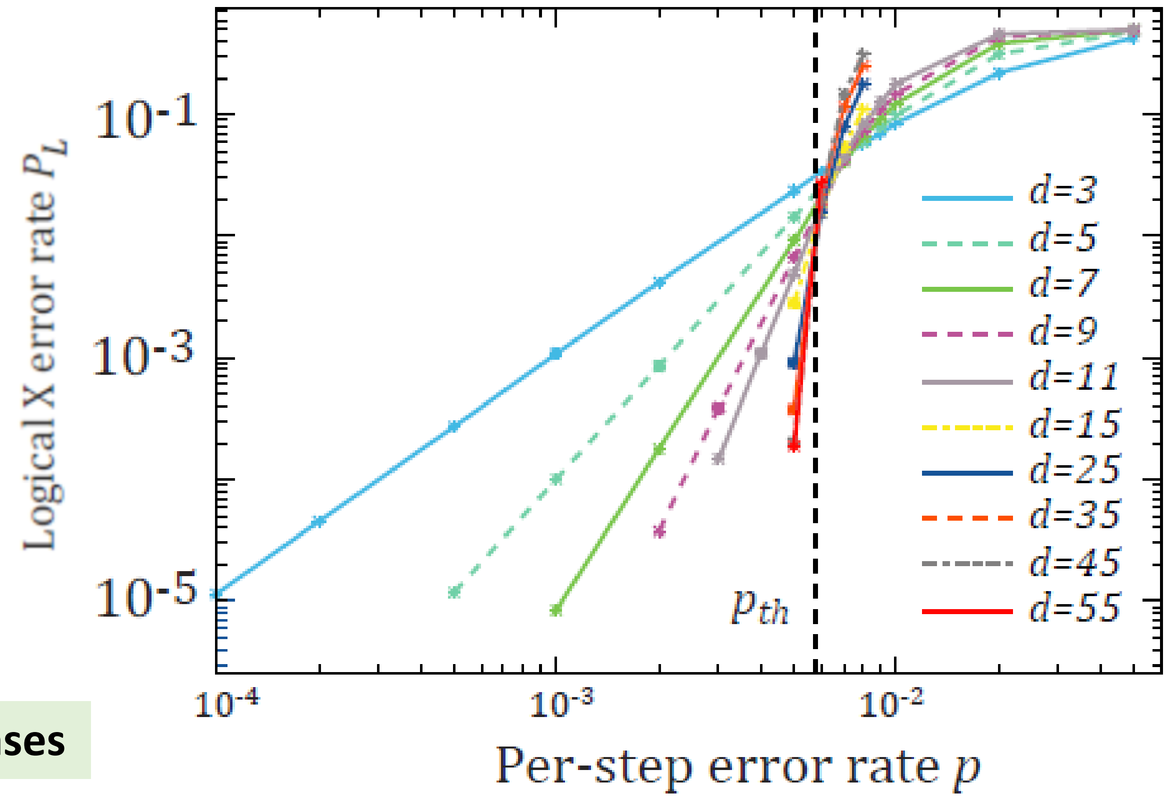
Logical Error P_L Versus Error Rate p

Relationship between P_L and p strongly depends upon the array size d

d (also called the distance) is the minimum number of physical qubit, bit-flip or phase flip needed to define $\widehat{X}_L$ or $\widehat{Z}_L$ operator

$$P_L \cong 0.03(p/p_{th})^{d_e}$$

$$(\boldsymbol{d_e} = (\mathrm{d}+1)/2$$



For all p (less than p_{th}), P_L is small, and gets smaller as d increases

For all p (greater than pth), P_L is larger, and gets larger as d increases

For $p < p_{th}$, the logical error rate (P_L) falls exponentially with d, while for $p > p_{th}$ P_L increases with d

Statistical Model for the Logical Error Rate

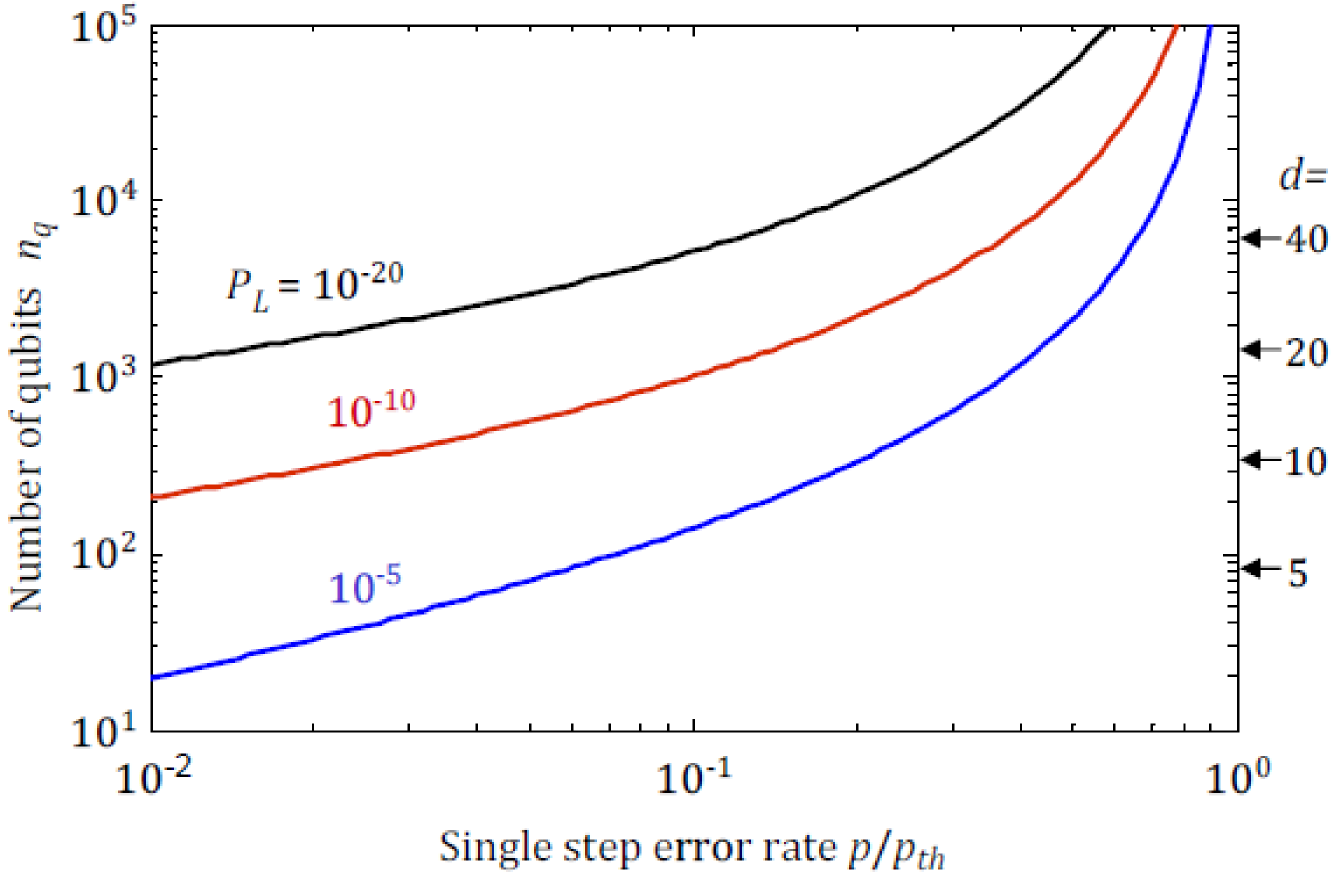
$$P_L^8 \cong d\frac{d!}{(d_e-1)!\,d_e!}(p_e)^{d_e}$$

$p_e = 8p$
8 steps per cycle

n_q increases rapidly as p approaches the p_{th}, a good target for the gate fidelity is above about 99.9 ($p \le 10^{-3}$)

Therefore, in this case logical qubits will need to contain $10^3 - 10^4$ physical qubits in order to achieve logical error rate below $10^{-14} - 10^{-15}$

Number of physical qubits n_q **per logical physical qubits** vs single step error rate p



Recent Experimental Results about Repeated/ Continuous QEC

The fault-tolerant operation of a quantum computer requires repeated detection and correction of both bit- and phase flip errors on data qubits

Next I will discuss repeated quantum error correction using the surface code and others, which is known for its exceptionally high tolerance to errors

13 qubit distance-two logical qubit in surface code

17 qubit distance-three logical qubit in surface code built upon distance-two error

Repeated QEC in Surface Code

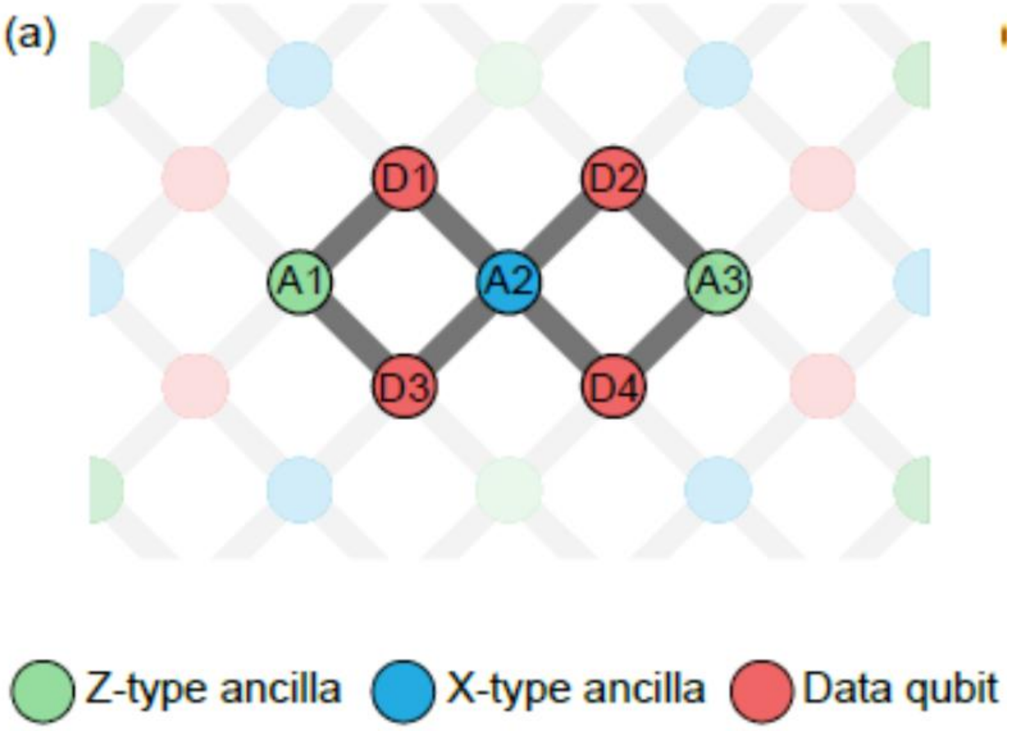
This paper experimentally implement its smallest viable instance, capable of repeatedly detecting any single error using seven superconducting qubits, four data qubits and three ancilla qubits.

C. K. Andersen, A. Remm, S. Lazar, S. Krinner, N. Lacroix, G. J. Norris, M. Gabureac, C. Eichler, and A. Wallraff, '***Repeated Quantum Error Detection in a Surface Code***', *Nat. Phys.* **16,** 875–880 (2020)

Seven Qubit Surface Code

Stabilizers of the distance-two surface code,

$$\hat{X}_{D1}\hat{X}_{D2}\hat{X}_{D3}\hat{X}_{D4}, \qquad \hat{Z}_{D1}\hat{Z}_{D3}\,, \qquad \hat{Z}_{D2}\hat{Z}_{D4}$$



Logical qubit Operators

$$Z_L = Z_{D1}Z_{D2}, \quad \text{or} \quad Z_L = Z_{D3}Z_{D4},$$
$$X_L = X_{D1}X_{D3}, \quad \text{or} \quad X_L = X_{D2}X_{D4},$$

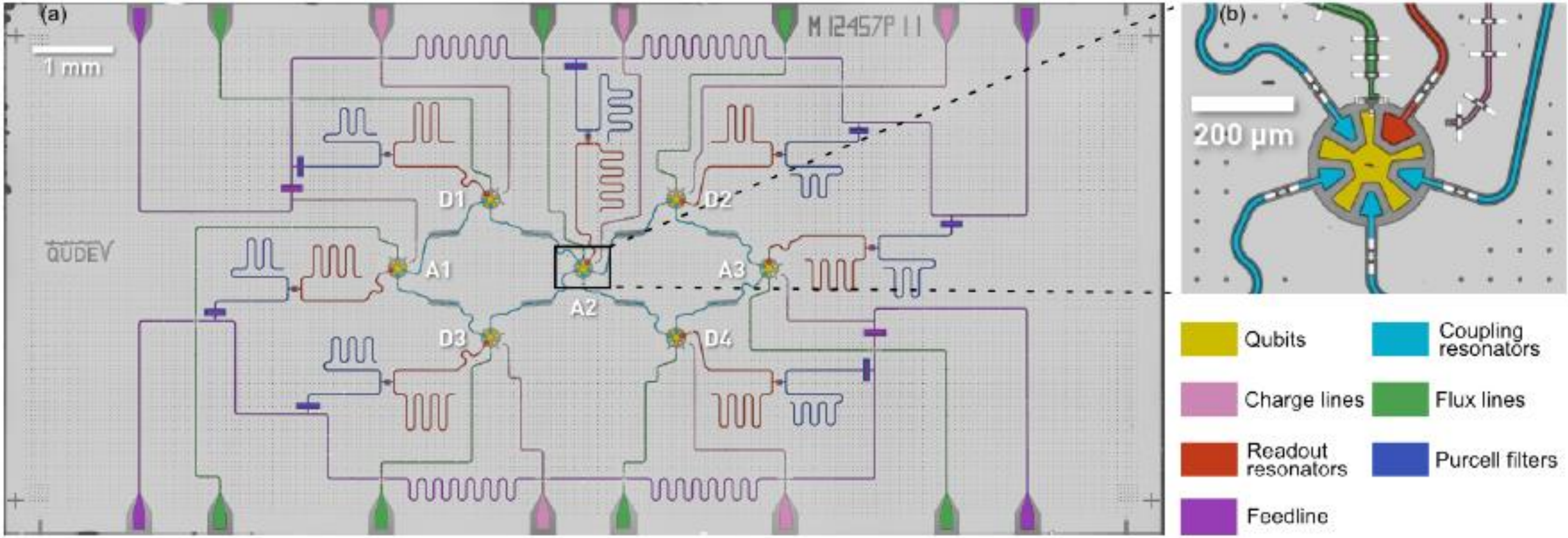
The surface code consist of a d x d grid of data qubits with d^2-1 ancilla qubit, each connected up to four data qubits. The code can detect d-1 error and correct up to [(d-1)/2] errors per cycle of stabilizer measurements.

For the code-distance d = 2, it is only possible to detect a single error per round of stabilizer measurements and once an error is detected, the error can not be unambiguously identified, e.g. one would obtain the same syndrome outcome for an X-error on D1 and on D3.

such that the code space in terms of the physical qubit states is spanned by the logical qubit states

$$|0\rangle_L = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle), \tag{4}$$
$$|1\rangle_L = \frac{1}{\sqrt{2}}(|0101\rangle + |1010\rangle). \tag{5}$$

Micrograph of the Seven Qubit QEC Device

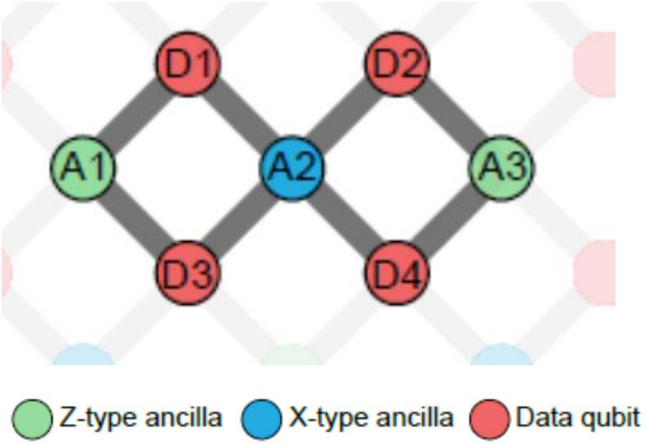


Transmon Qubits (yellow), flux line for frequency tuning (green), individual charge line (pink) for single qubit gate

Each qubit is coupled to readout resonator (red)

Gate Sequences of Quantum Error Detection

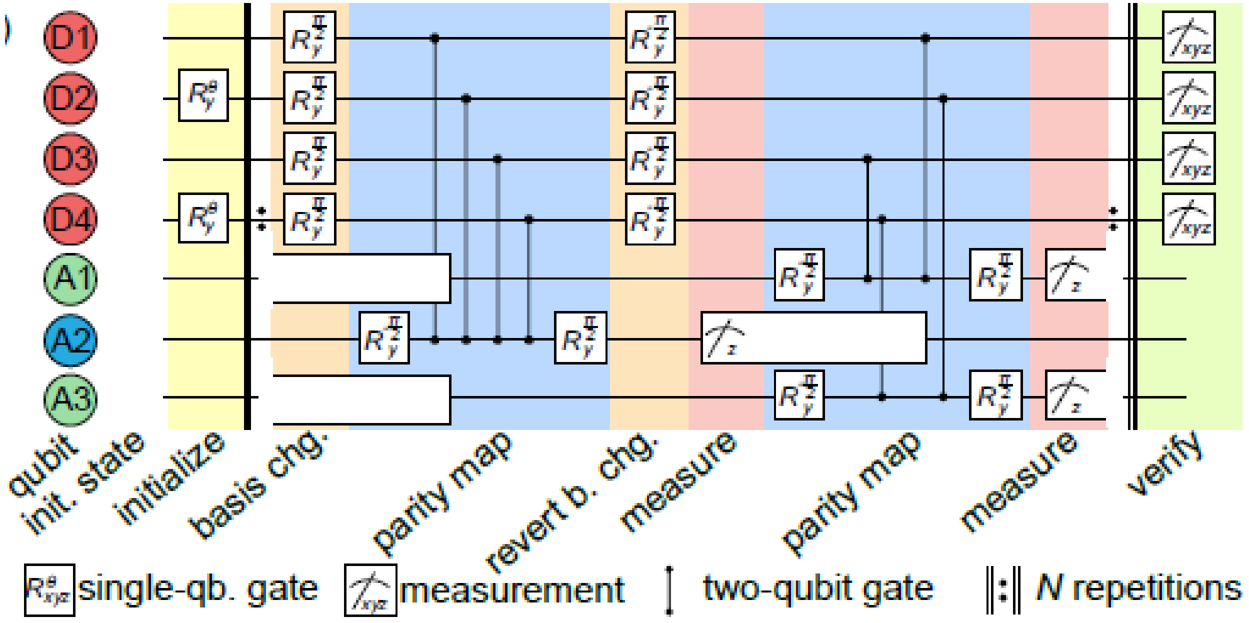
$\hat{X}_{D1}\hat{X}_{D2}\hat{X}_{D3}\hat{X}_{D4}$ stabilizer measurement by first applying basis change pulses ($R_Y^{\pi/2}$)on the data qubits to map the X basis to the Z-basis



Perform entangling gate and finally revert the basis change.

The measurement of A2 will therefore yield the $|0\rangle$-state ($|1\rangle$-state) corresponding to the eigenvalues +1 (-1) of the stabilizer $\hat{X}_{D1}\hat{X}_{D2}\hat{X}_{D3}\hat{X}_{D4}$.

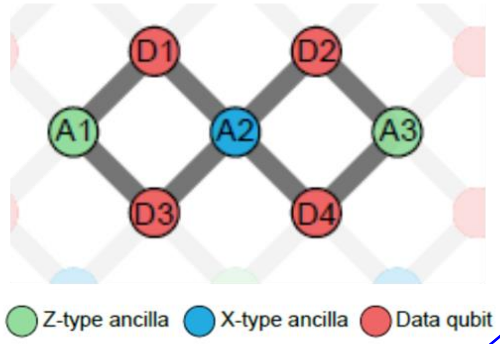
While measurement pulses for A2 is still being applied, they perform the $\hat{Z}_{D1}\hat{Z}_{D3}$ and $\hat{Z}_{D2}\hat{Z}_{D4}$ measurements simultaneously using the ancilla qubit A1 and A3, respect.



The cycle is repeated after above steps, N-times and state tomography was performed

Stabilizer Measurements of Data Qubit

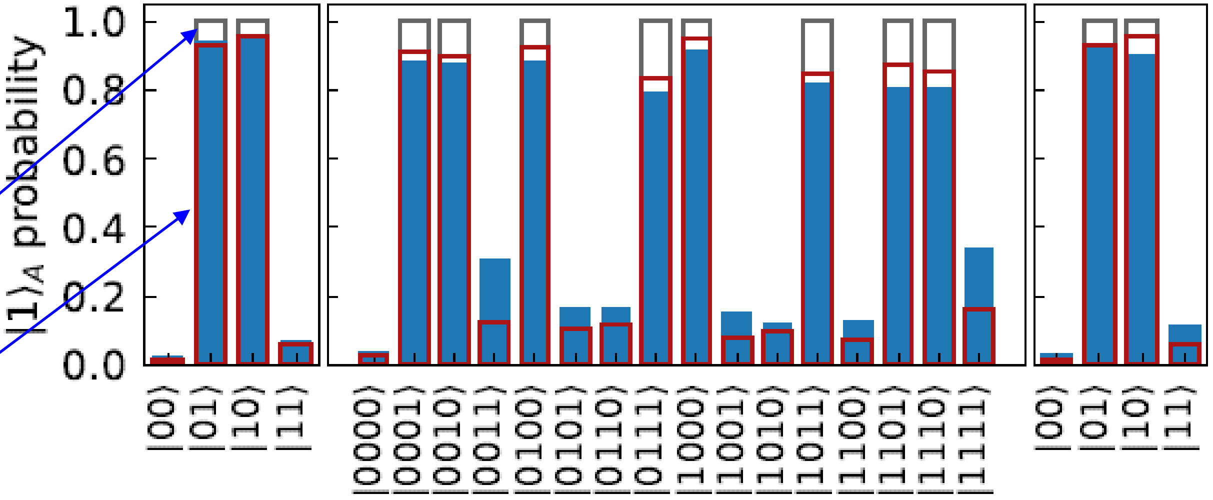
Changes in the outcome of repeated stabilizer measurements (Syndrome), signals the occurrence of errors



Preparing the data qubit in each of the computational basis states and measuring the Z-stabilizer

Gray-ideal case

Red – Simulation that includes decoherence & readout errors



	Init State, $|D_1D_3\rangle$	Init State, $|D_1D_2D_3D_4\rangle$	Init State, $|D_2D_4\rangle$
Stabilizer	$\hat{Z}_{D1}\hat{Z}_{D3}$	$\hat{Z}_{D1}\hat{Z}_{D2}\ \hat{Z}_{D3}\hat{Z}_{D4}$	$\hat{Z}_{D2}\hat{Z}_{D4}$
using ancilla	A_1	A_2	A_3
Success Probabilities of Stabilizers	95.5 %	83.5 %	91.8 %

It is critical to directly verify the ability to measure multi qubit stabilizers with ancilla

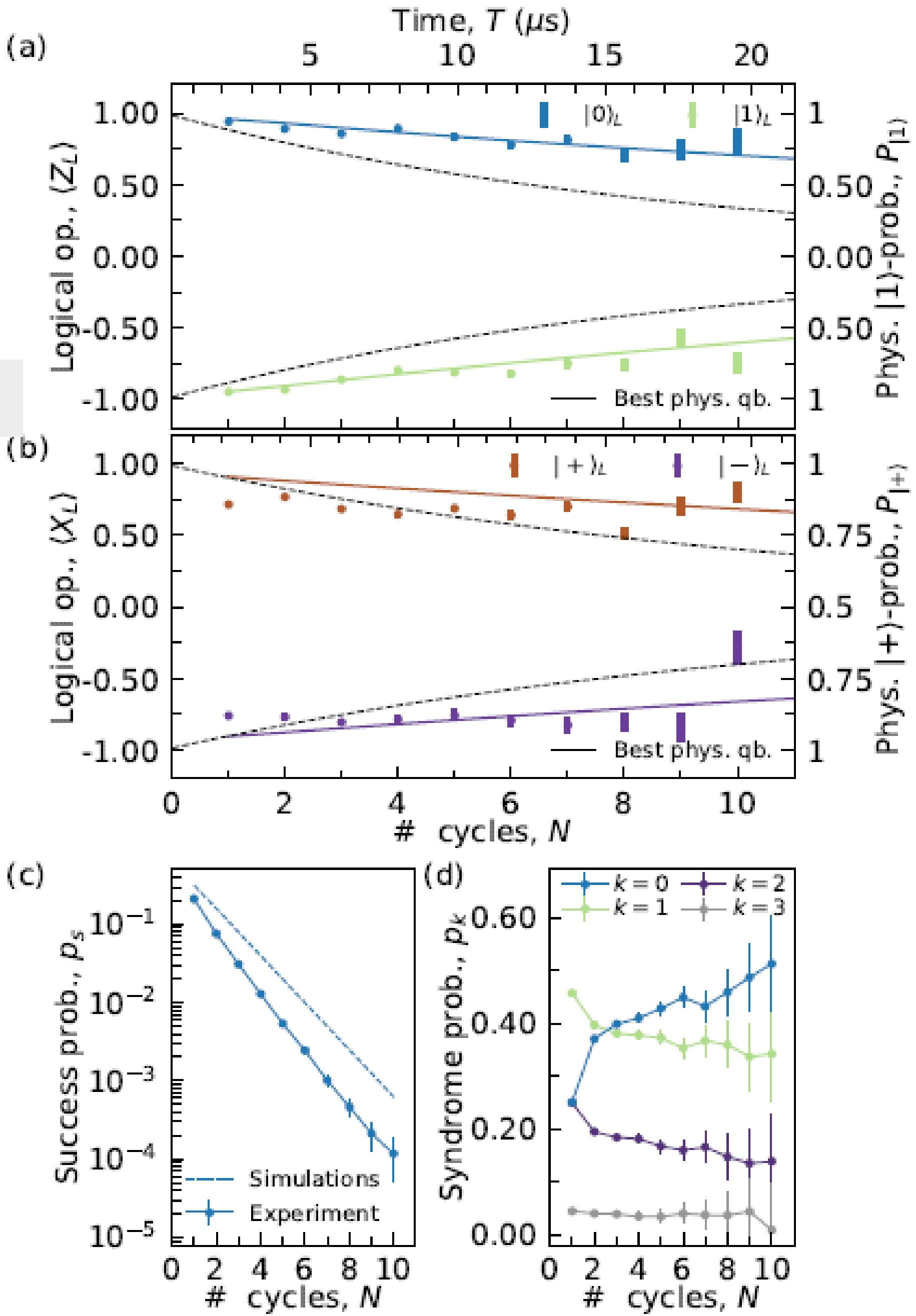
Parity measurements mainly limited by the relaxation of qubit, worst case as $|1111\rangle$

Repeated Quantum Error Detection

Experimentally determined and simulated expectation value of Z_L

Solid line are simulation-based

Experimentally determined and simulated expectation value of X_L



Distance-Three Surface Code in Superconducting Circuits

They demonstrated quantum error correction using the surface code, which is known for its exceptionally high tolerance to errors

Using 17 physical qubits in a superconducting circuit, **encoded quantum information in a distance-three logical qubit** building up on recent distance-two error detection experiments

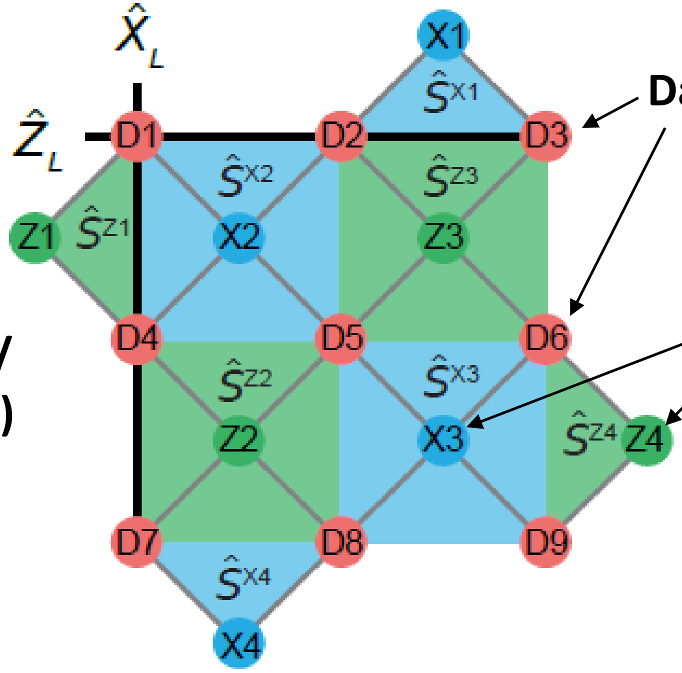
In an error correction cycle takes only 1.1 μs, demonstrated the preservation of four cardinal states of the logical qubit.

Repeatedly executed the cycle, **measure and decode both bit- and phase-flip error syndromes** using a minimum-weight perfect-matching algorithm in an error-model free approach and apply corrections in postprocessing.

S. Krinner, N. Lacroix, Ants Remm, Agustin Di Paolo, E. Genois, C. Leroux, C. Hellings, S. Lazar, F. Swiadek, J. Herrmann, G. J. Norris, C. K. Andersen, M. Müller, A. Blais, C. Eichler & A. Wallraff 'Distance-Three Surface Code in Superconducting Circuits' *Nature* **volume 605**, pages669–674 (2022)

Distance-Three Surface Code in Superconducting Circuits

Experimentally realizing a **distance-three surface code requires nine data qubits and eight auxiliary qubits** (ancilla or measurement qubits

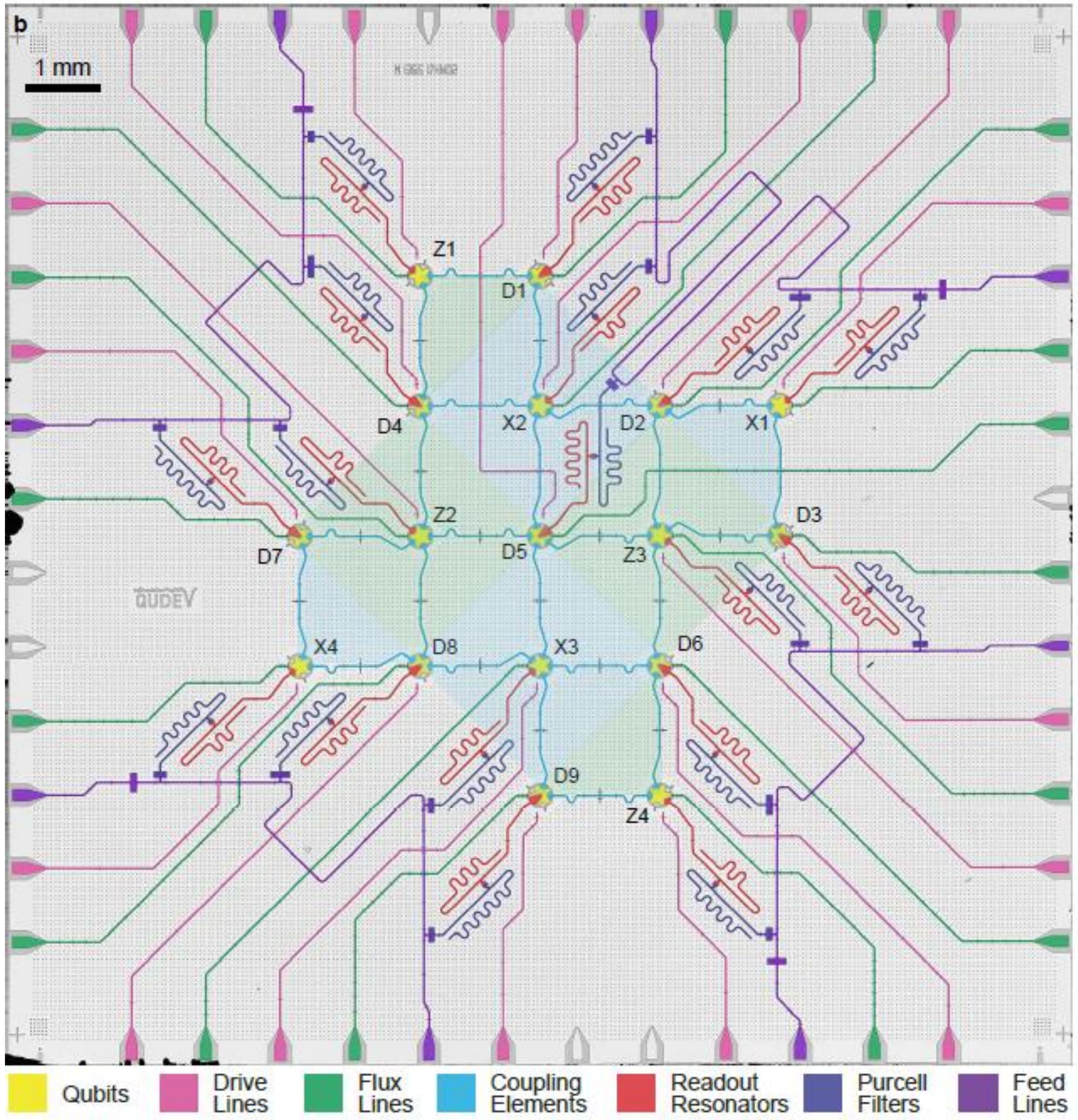


Data Qubits (red)

Ancilla Qubits (green/blue)

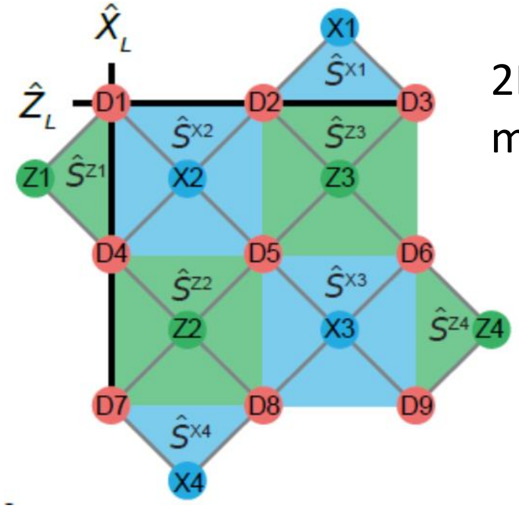
X-type (blue)/ Z-type (green) stabilizer

Data Qubits in the **weight-three logical** operators $\hat{Z}_L$ and $\hat{X}_L$ (solid black line)

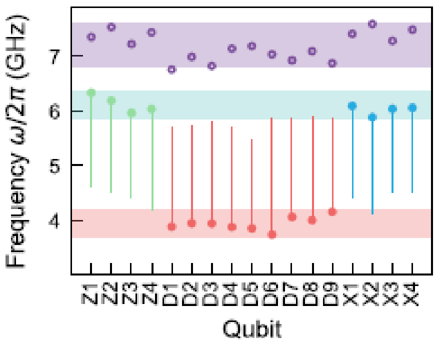


False-color micrograph of the device realizing the concept with 17 transmon

Stabilizer Circuits and Data Qubit Measurements

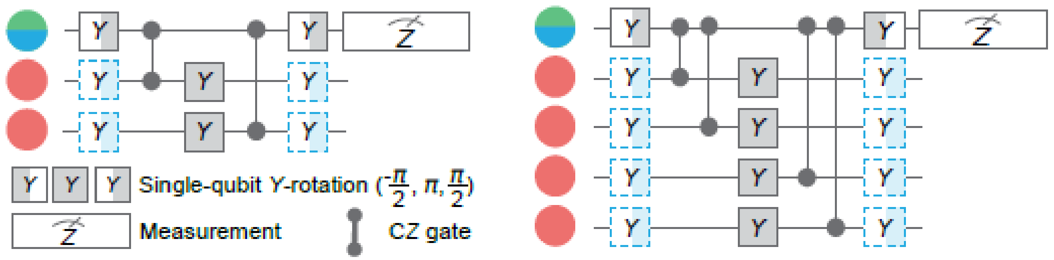


2D array of data and ancilla/ measurement qubits



Different frequencies for Data and CZ (2/4)

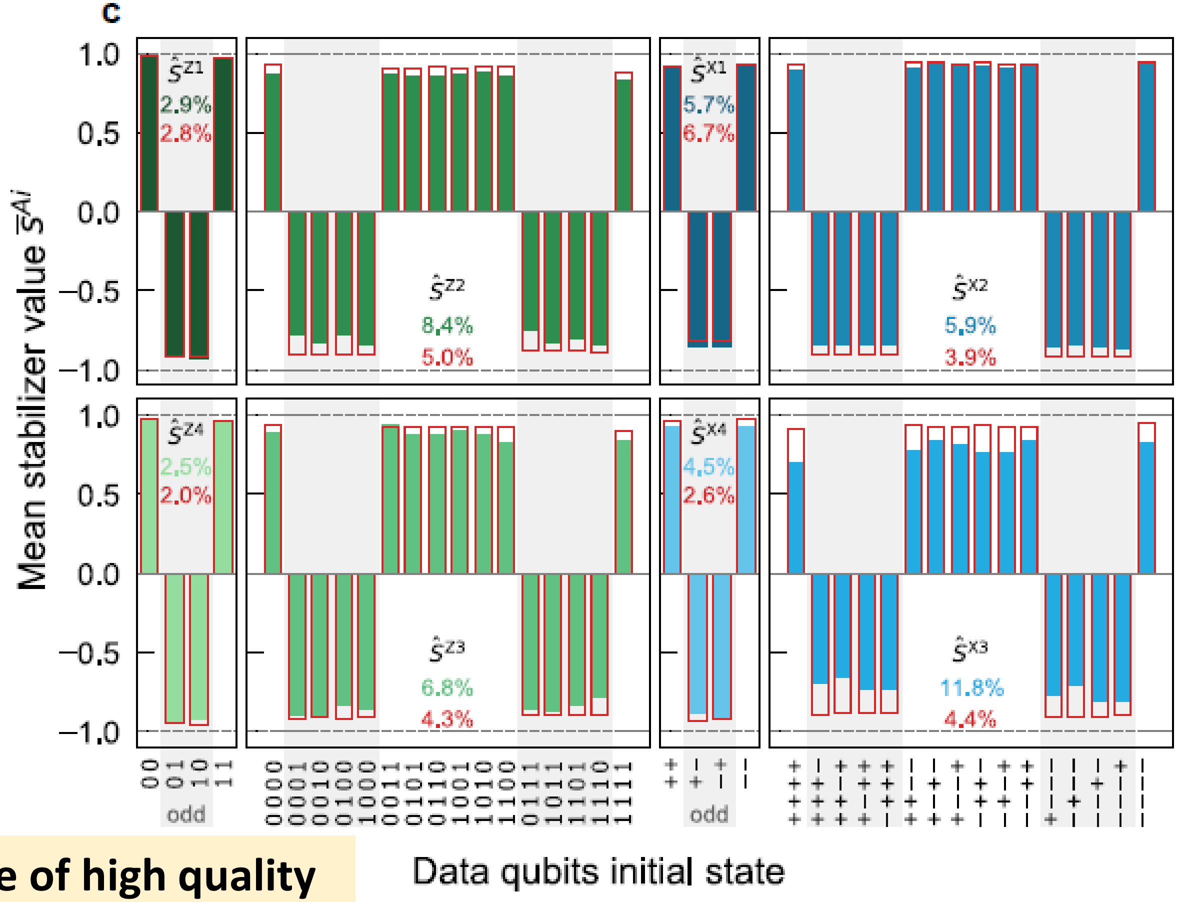
Weight-two **weight-four** stabilizer circuit



Average stabilizer value
Measured - filled bars
Simulation - red boundary

White regions even parity
Gray regions odd parity

Percentage error
Blue/ green measured
Red - simulated



All stabilizer measurements are of high quality

Surface Code Cycle

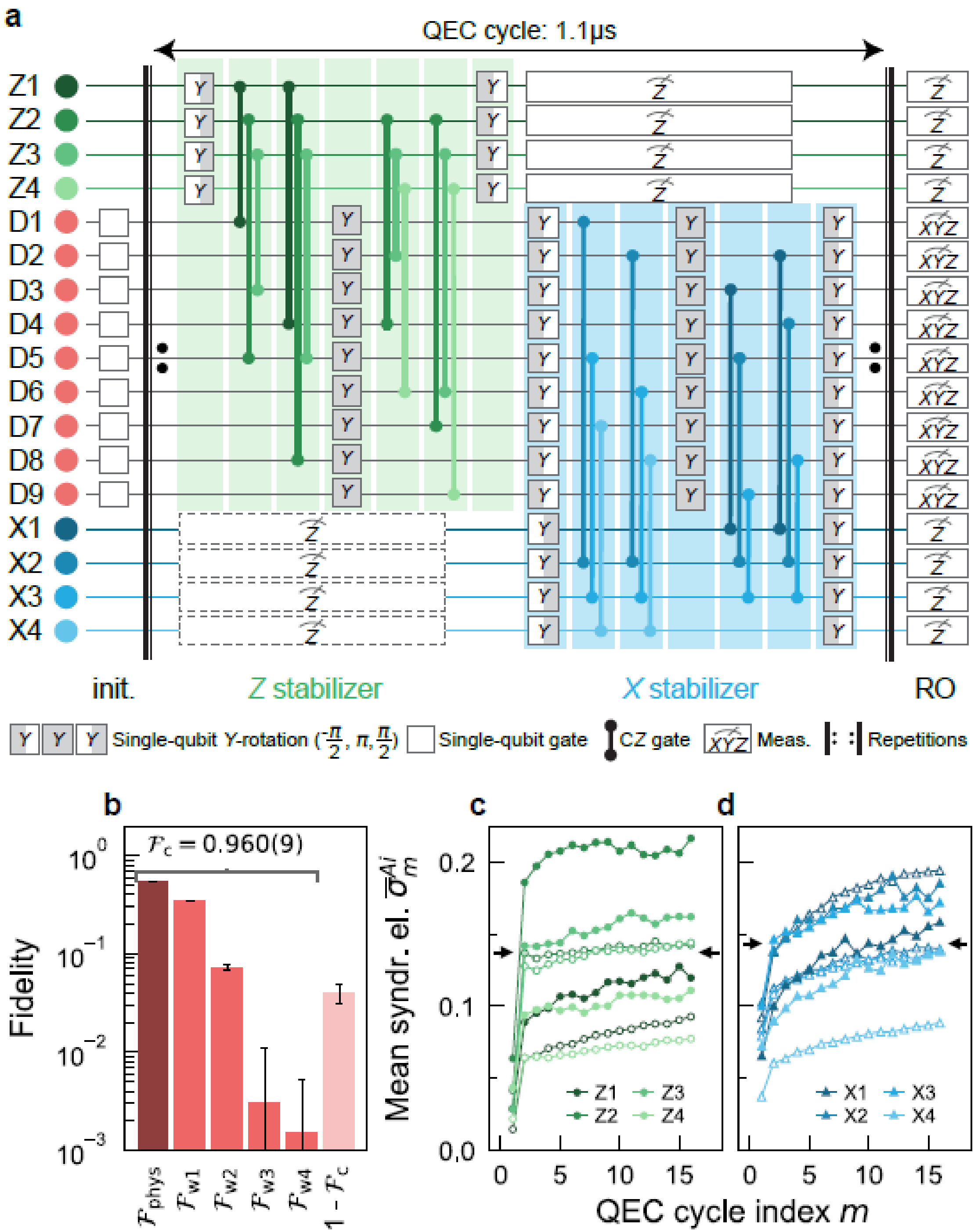
Simultaneous Z-type and x-type auxiliary qubit readout
(All Z- and X- stabilizer measurements in parallel)

Shortest single QEC cycle 1.1 μ s

Fidelity of prepared initial state (including SC correction)

$$\mathcal{F}_c = \mathcal{F}_{phys} + \sum_{i=1}^{4} \mathcal{F}_{wi} = 96.0\%$$

Logical error P_L(=54.2%) and $\mathcal{F}_{phys}$ (54.0 %)
are smaller than in distance-two surface code
(expected because increase in distance)



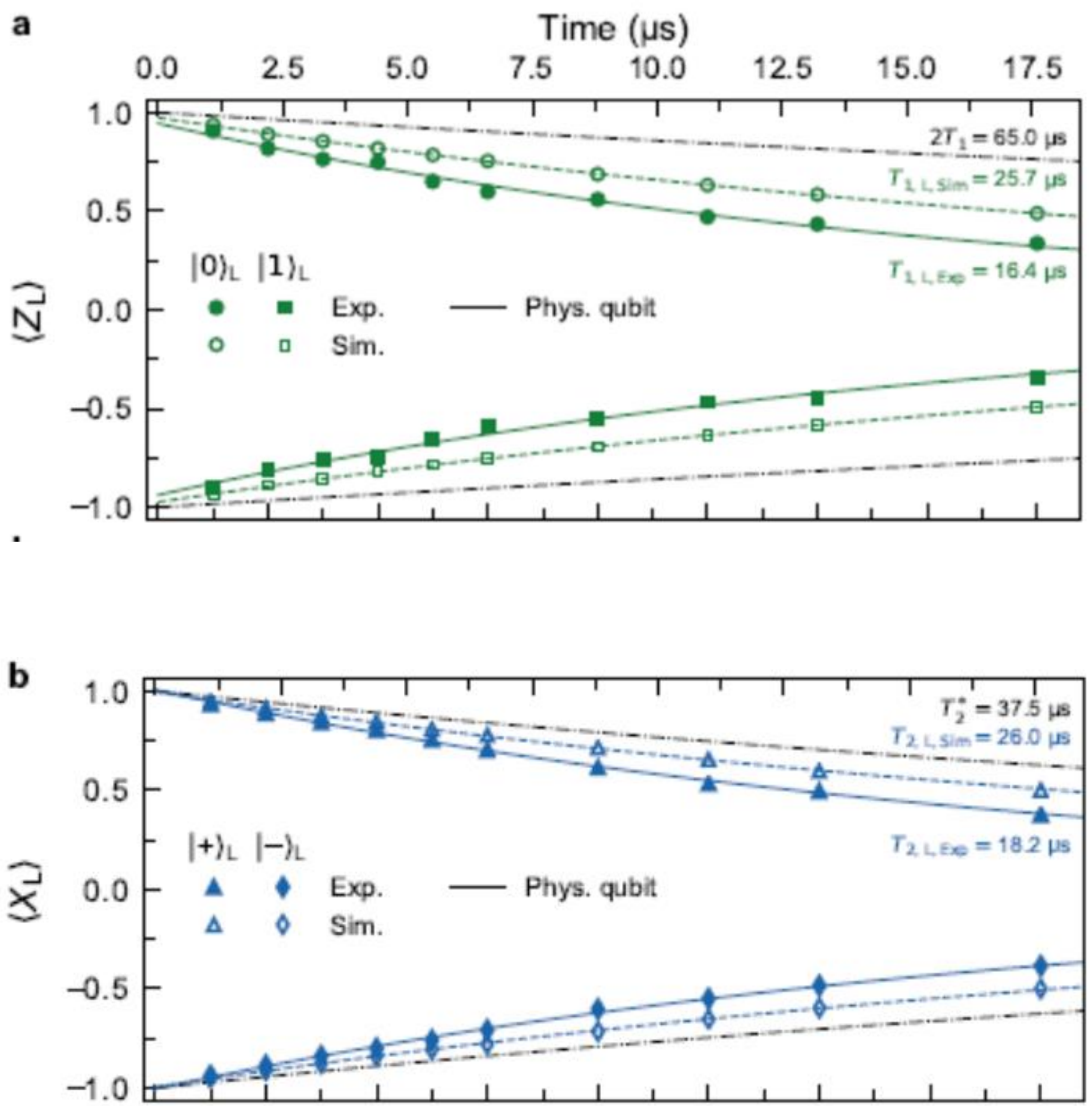
Logical State Preservation and Error per Cycle

Experimentally determined and simulated expectation value of Z_L

The decay time is much more than the QEC cycle (1.1 μs)

Experimentally determined and simulated expectation value of X_L

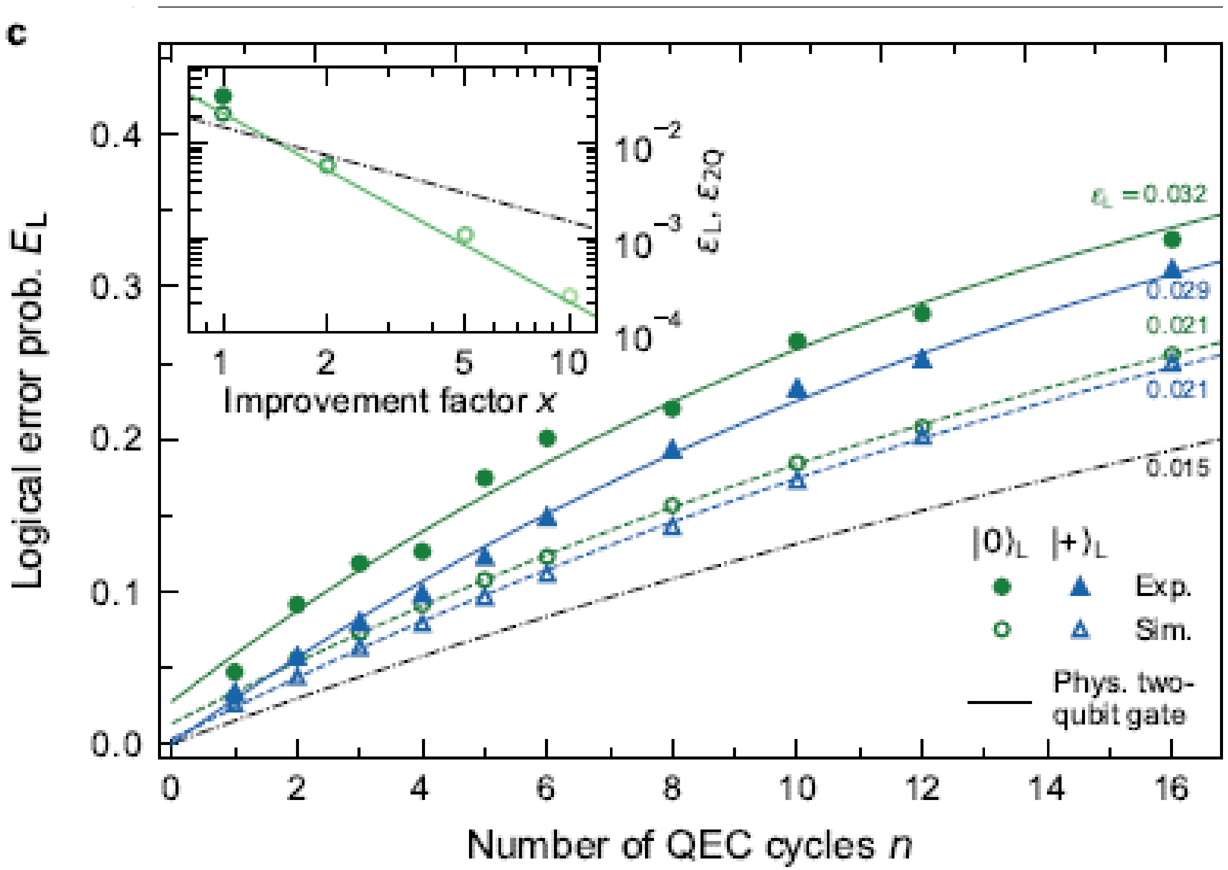
The decay time is much more than the QEC cycle (1.1 μs)



Logical State Preservation and Error per Cycle

Logical error probability E_L as a function of number of error correction cycle (extracted from expectation value of Z/X) and extracted error per cycle ϵ_L

Coherence time $T_{2,L}$ = 18.2 μs and life time $T_{1,L}$ = 16.4 μs of the logical qubit
(extracted from decay curve of $\langle \hat{Z}_L \rangle$ and $\langle \hat{X}_L \rangle$) are much larger than QEC cycle)



Experimental Demonstration of Continuous QEC

Typically, quantum error correction is **executed in discrete rounds**, using entangling gates and projective measurement on ancillary qubits to complete each round of error correction.

They used **direct parity measurements** to implement a **continuous quantum bit-flip correction code** in a resource-efficient manner, eliminating entangling gates, ancillary qubits, and their associated errors.

Furthermore, **the protocol increases the relaxation time of the protected logical qubit by a factor of 2.7** over the relaxation times of the bare qubits.

Results showcase **resource-efficient stabilizer measurements in a multi-qubit architecture** and demonstrate how continuous error correction codes can address challenges in realizing a fault-tolerant system.

W. P. Livingston, M. S. Blok, E. Flurin, J. Dressel, A. N. Jordan & Irfan Siddiqi, 'Experimental Demonstration of Continuous QEC', NATURE COMMUNICATIONS (2022) 13:2307

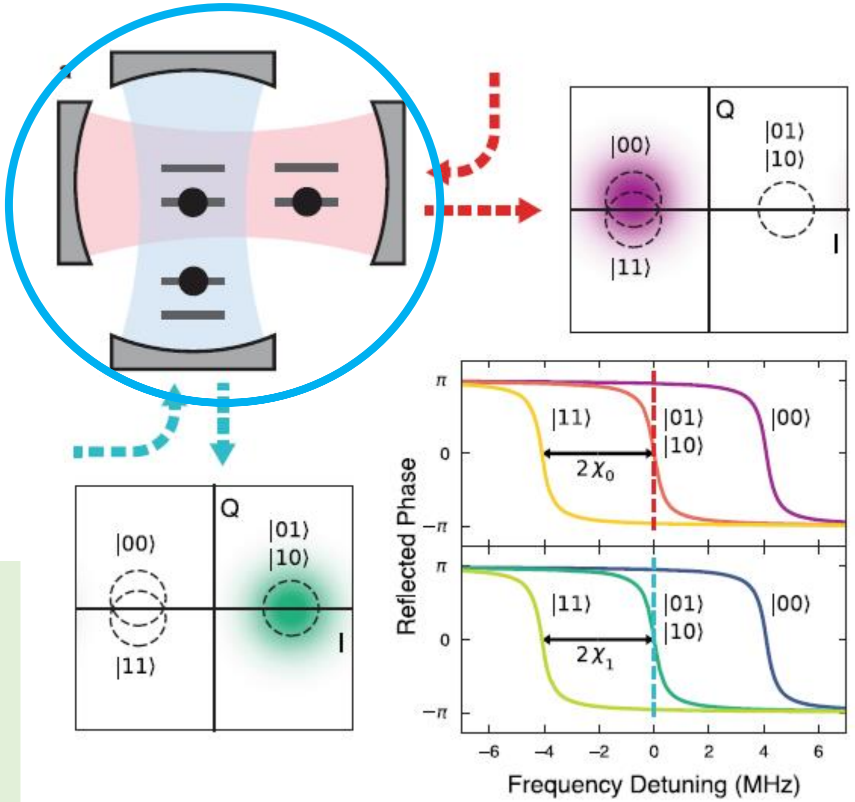
Parity Measurements of Two-Qubits in one Cavity

ZZ parity measurements using two pairs of qubits coupled to joint readout resonator (with same dispersive coupling

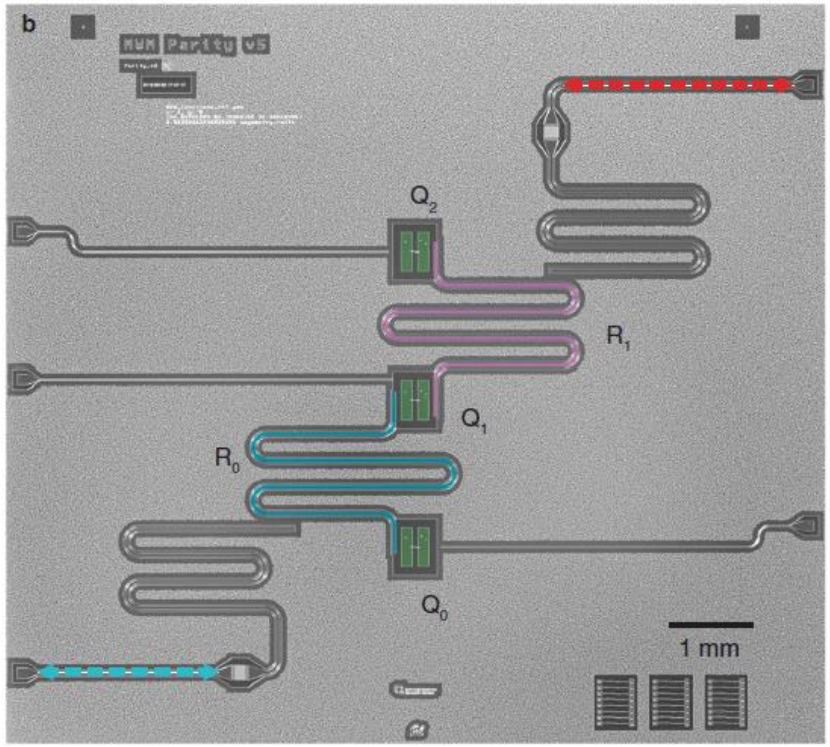
Qubits measurement, parity response in state $|01\rangle$ identical to state $|10\rangle$

When Qubit pair is in either $|00\rangle$ or $|11\rangle$, the resonance frequency is far detuned from the odd parity measurements

Basic Idea



Implementation

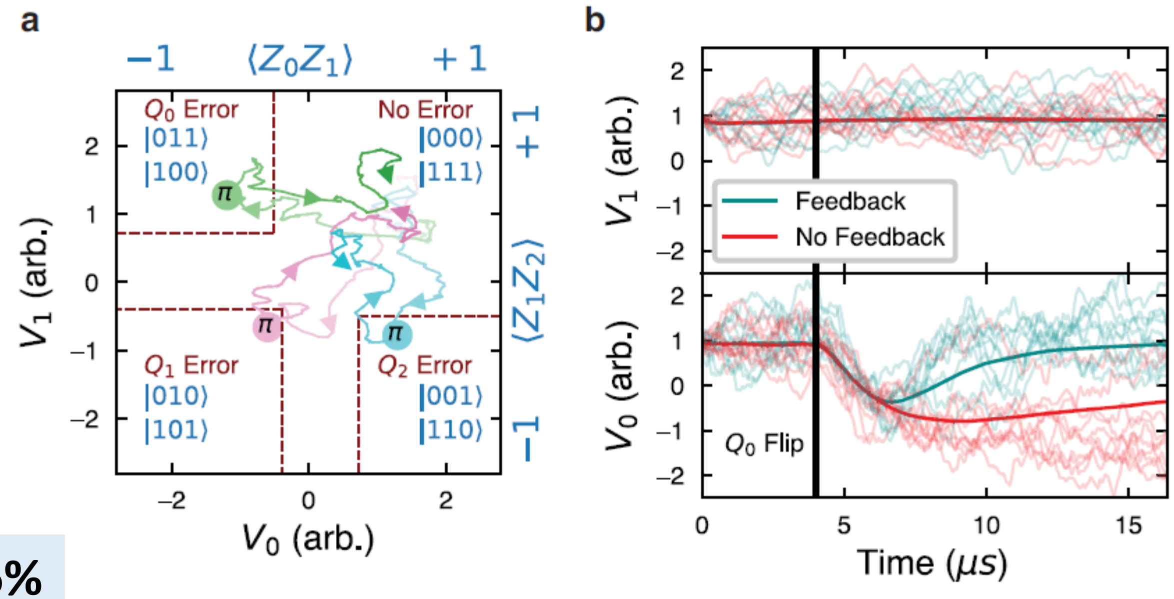


Z-parity stabilizers: Z_0Z_1 and $Z_1\,Z_2$ With Logical state $|0_L\rangle = |010\rangle$ and $|1_L\rangle = |101\rangle$ With parity value (-1, -1)

Any other parity value will detect the error

Error Detection and Correction

First they created the error delicately and successfully tuned and check the reliability of measurement setup



Errors detected on Q_0 with 90% efficiency, Q_1 with 86% efficiency and Q_2 with 91% efficiency (mainly due to T1 decay of qubits, error correction time 3.1 – 3.4 μs)

Using Z-parity stabilizers, Z_0Z_1 and Z_1Z_2errors were successfully detected and corrected as well

Logical qubit excited life 2.7 times the longer than of the bare physical qubit was observed

They studied few more interesting paraments related to this new continuous quantum error detection and correction

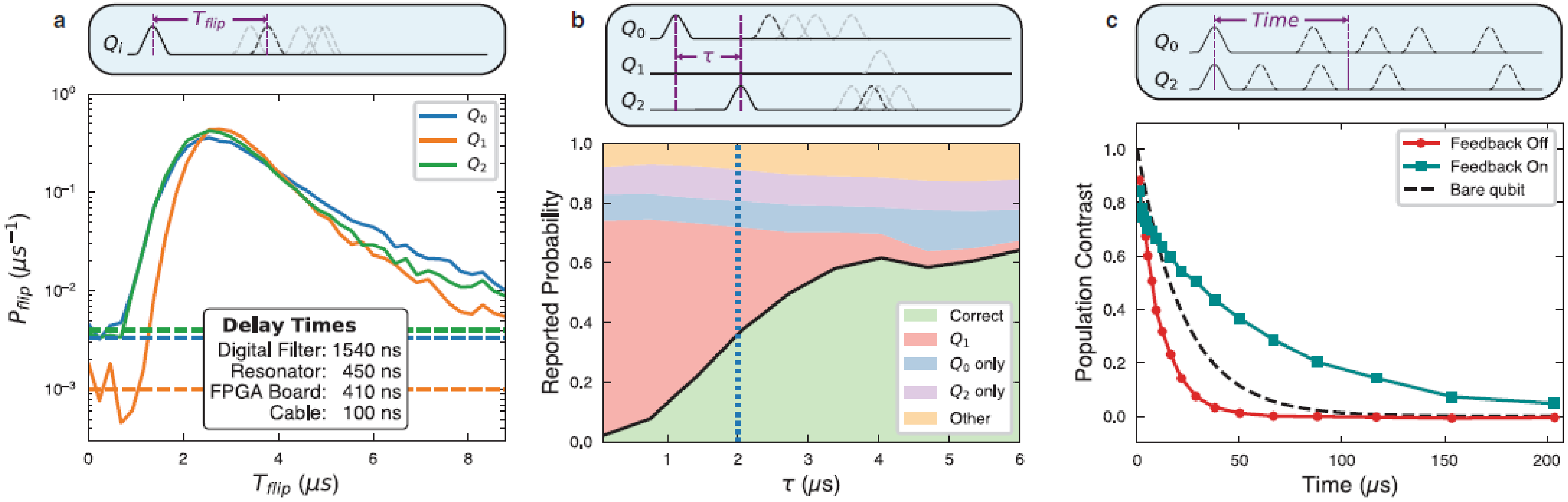
Conclusion

- **Surface code is very promising model for complete QEC**

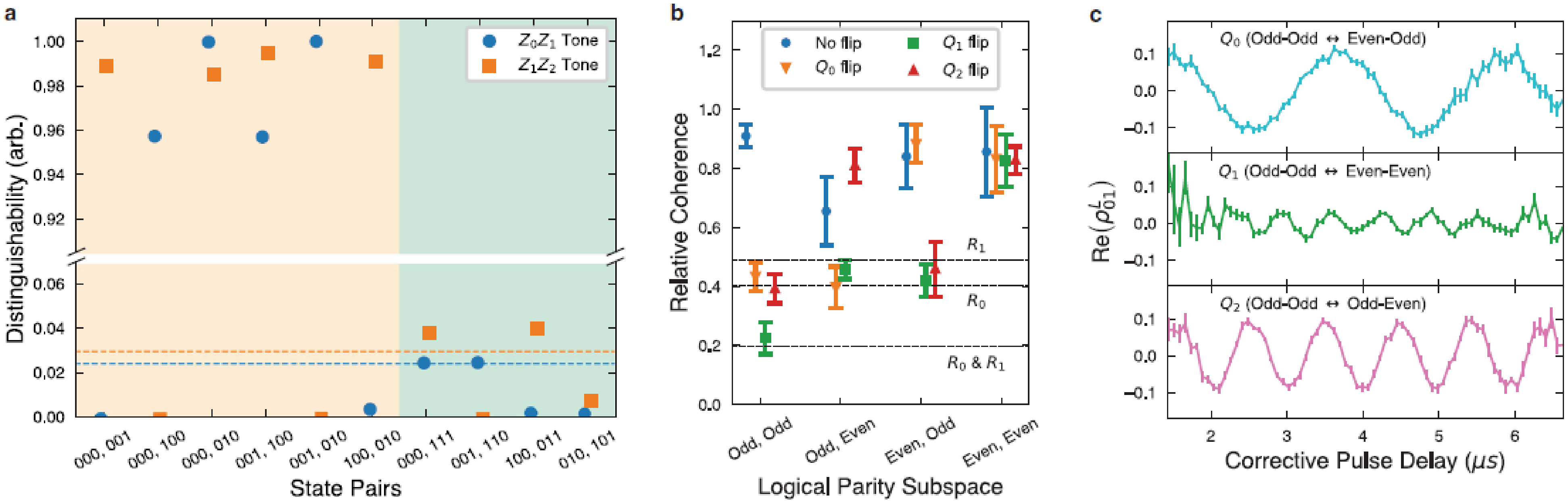
- **Surface code can be executed experimentally but with limited number of qubits**

- **Complexity of circuits increase as the number of qubit increases**

- **Bit flip error can be continuously detected and corrected without complex surface code circuitry**



a
b
c
T_{flip}
Q_i
P_{flip} (μs^{-1})
T_{flip} (μs)
Delay Times
Digital Filter: 1540 ns
Resonator: 450 ns
FPGA Board: 410 ns
Cable: 100 ns
Q_0
Q_1
Q_2
τ
Reported Probability
τ (μs)
Correct
Q_1
Q_0 only
Q_2 only
Other
Time
Population Contrast
Time (μs)
Feedback Off
Feedback On
Bare qubit

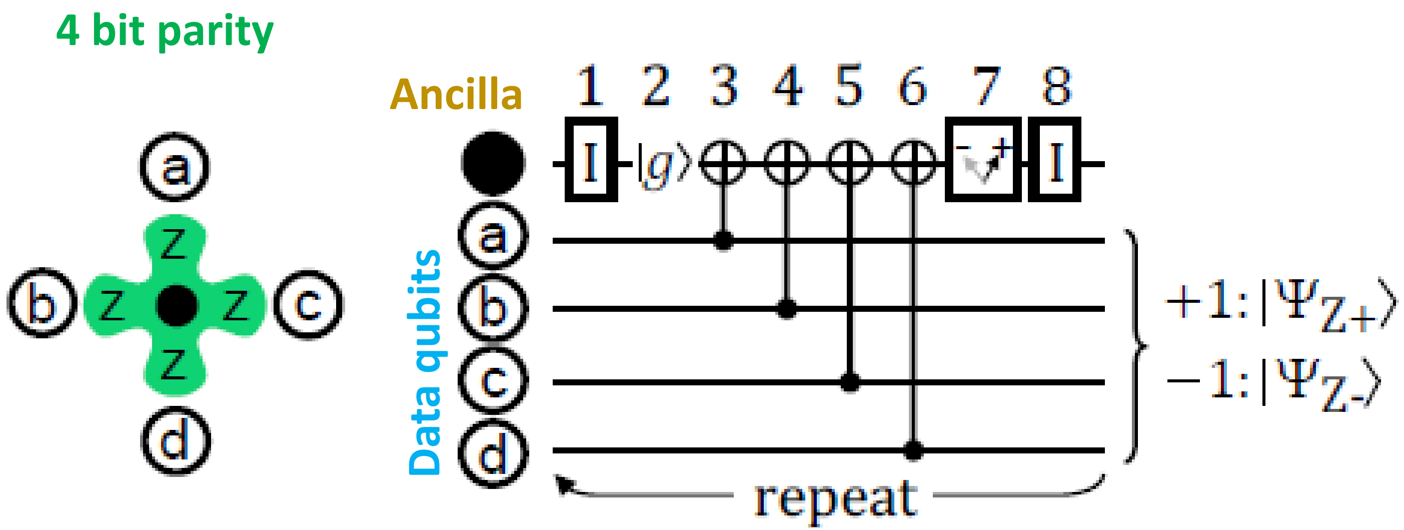

a
b
c
Z_0Z_1 Tone
Z_1Z_2 Tone
Distinguishability (arb.)
State Pairs
000, 001
000, 100
000, 010
001, 100
001, 010
100, 010
000, 111
001, 110
100, 011
010, 101
No flip
Q_0 flip
Q_1 flip
Q_2 flip
Relative Coherence
Logical Parity Subspace
Odd, Odd
Odd, Even
Even, Odd
Even, Even
R_1
R_0
R_0 & R_1
Q_0 (Odd-Odd ↔ Even-Odd)
Q_1 (Odd-Odd ↔ Even-Even)
Q_2 (Odd-Odd ↔ Odd-Even)
$\mathrm{Re}(\rho^L_{01})$
Corrective Pulse Delay (μs)

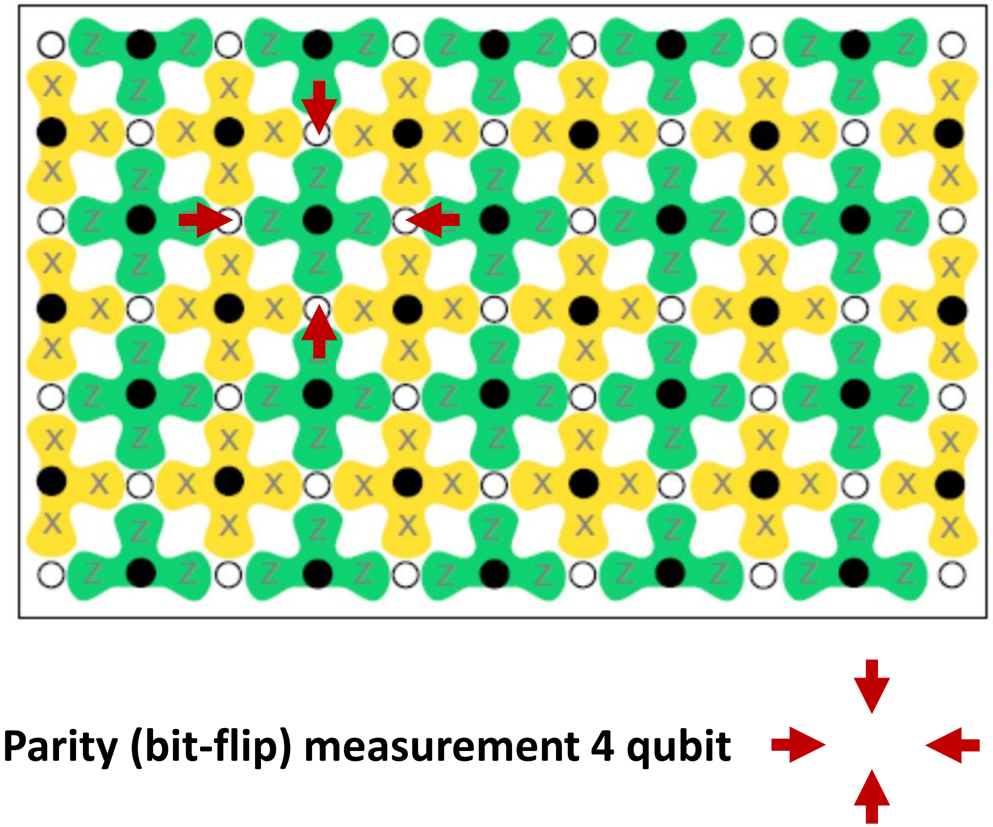
In the context of QEC, a qubit is generally not a two-level system, where two states are eigenstates of the Hamiltonian and transitions between the two states can easily be driven. For example, it can be an ensemble of two-level systems or, in this thesis, some states of a harmonic oscillator.

The goal of quantum error correction is to create logical
qubits that are less noisy than the physical qubits that are available. For this, the information of a quantum bit needs to be encoded such that it is hidden from the environment and can be decoded later without errors.

Surface Code-Bit Flip Measurements

Circuit to measure the bit-flip error





Parity (bit-flip) measurement 4 qubit

If any of the four bit (a, b, c, d) is flipped (parity measurements) measurements out outcome is −1

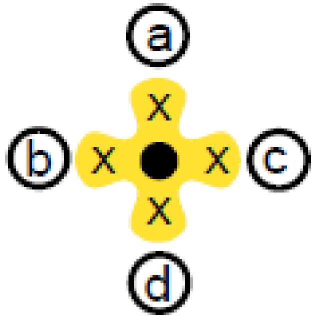
Which one is flipped ?

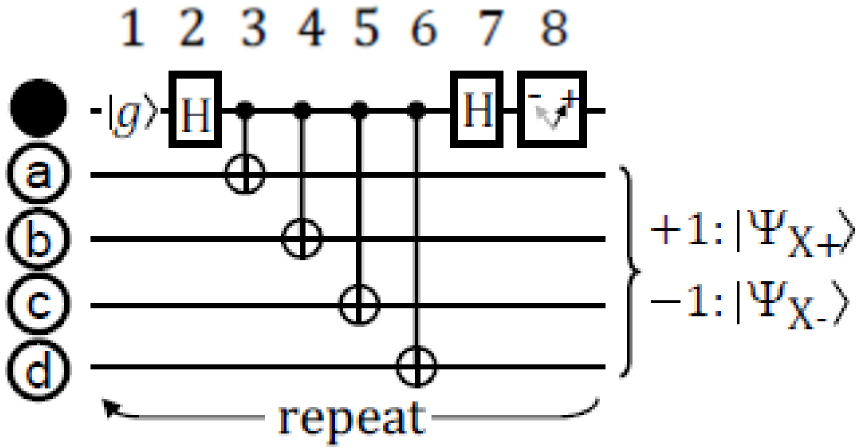
We don't know

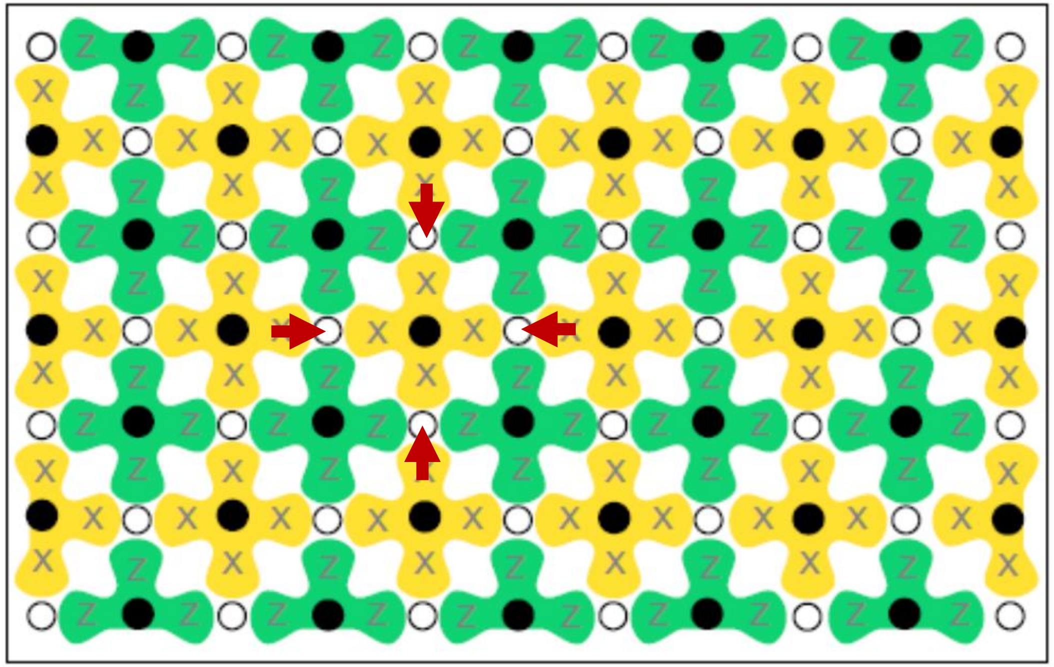
Surface Code – Phase flip Measurements

Circuit to measure the phase-flip error

4 phase parity

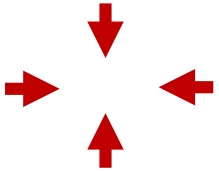






If any of the four bit (a, b, c, d) is flipped (parity measurements) measurements out outcome is –1

Parity (phase) measurement 4 qubit

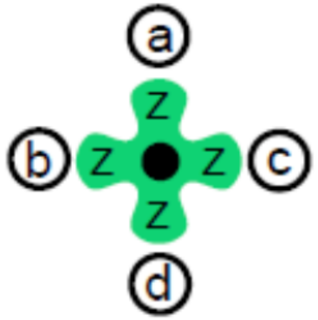


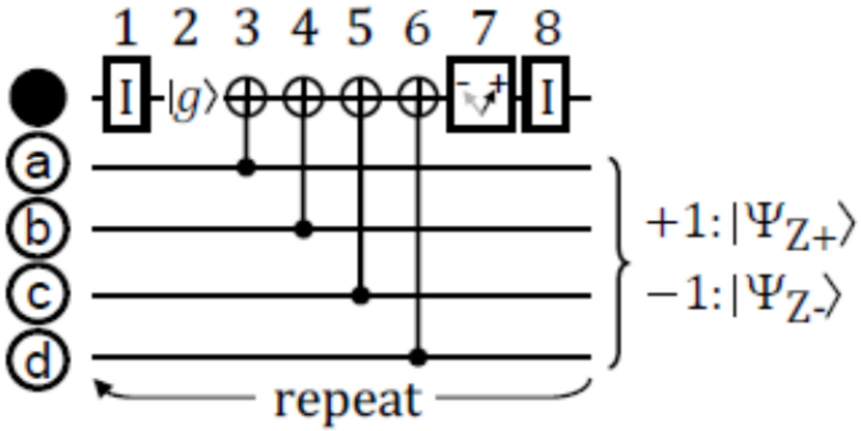
Which one is flipped ?

We don't know

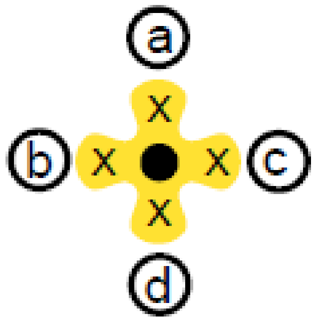
Surface Code

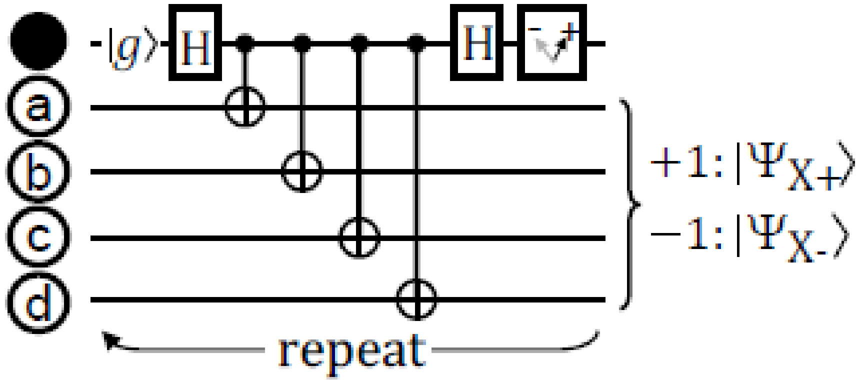
4 bit parity

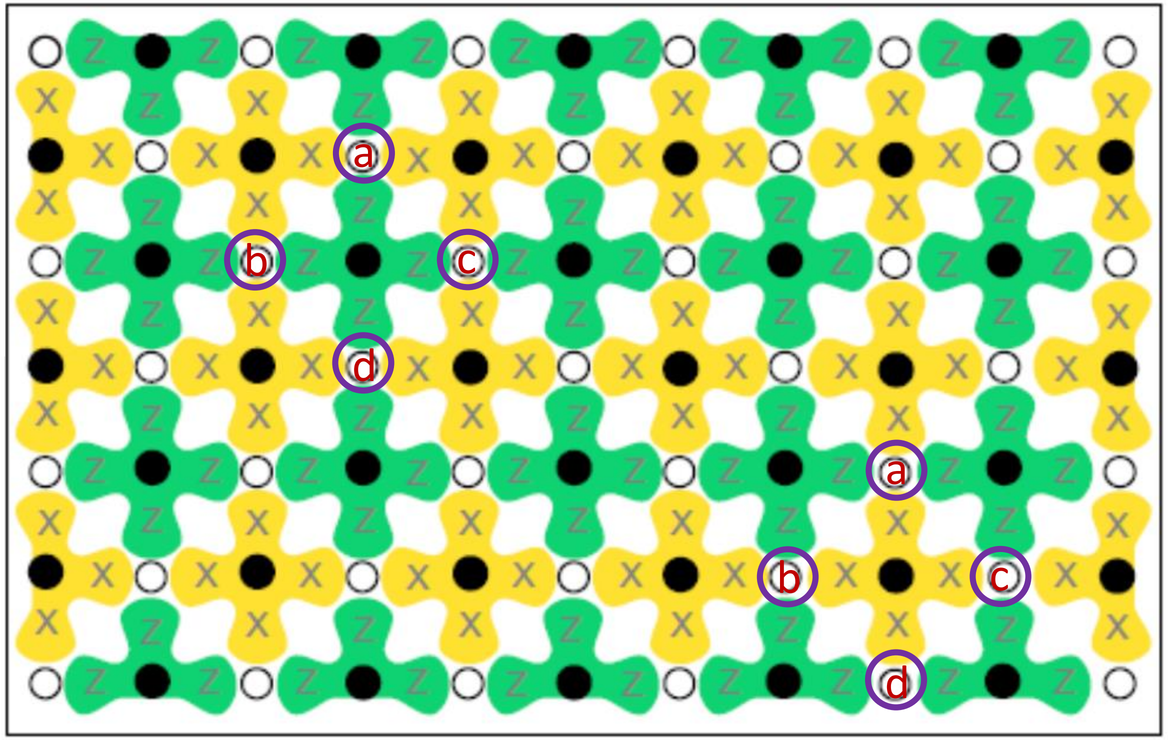




4 phase parity







Recent Experimental Results about Repeated/ Continuous QEC

The fault-tolerant operation of a quantum computer requires repeated detection and correction of both bit- and phase flip errors on data qubits

For fault-tolerant operation quantum computers must correct errors
occurring due to unavoidable decoherence and limited control accuracy

Here, we demonstrate quantum error correction using the surface code,
which is known for its exceptionally high tolerance to errors [3–6]

Using 17 physical qubits in a superconducting circuit we encode quantum information in a
distance-three logical qubit building up on recent distance-two error detection experiments [79].

Realizing Repeated Quantum Error Correction in a Distance-Three Surface Code

In an error correction cycle taking only 1.1μs, we demonstrate
the preservation of four cardinal states of the logical qubit

Surface Code

One approach to building a quantum computer is based on surface code

Realization of a 'surface code logical qubit' is key goal for many quantum computing hardware efforts

The challenge in creating quantum error correction codes lies in finding commuting sets of stabilizers that enable errors to be detected without disturbing the encoded information.

In terms of actual implementation, the specific **advantage of surface code for current hardware platforms** is that it requires only **nearest-neighbor interactions**. *(Short/long Interaction, with high fidelity, strongly depends upon hardware platform)*

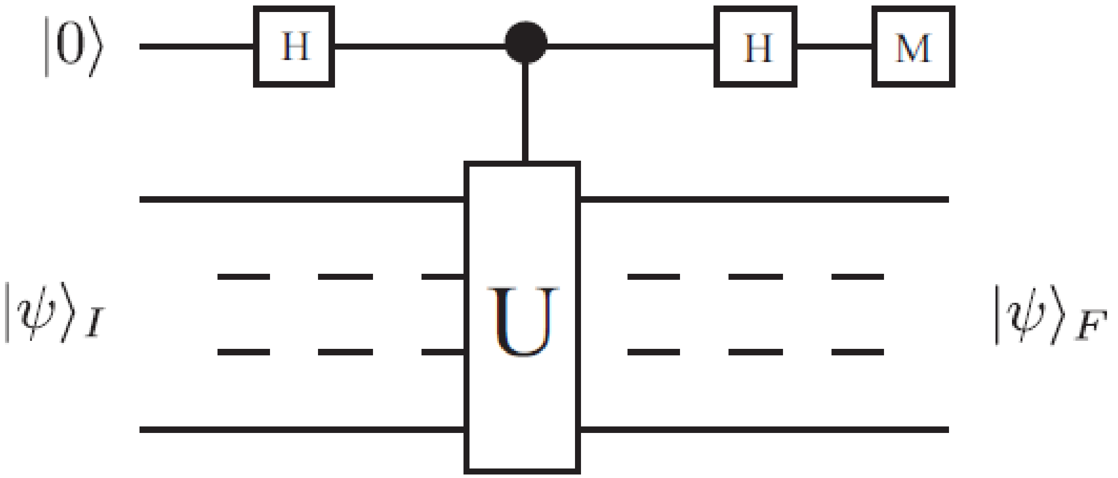
If we detect all the errors, we can correct them by repeatedly applying quantum correction gates. However, **one feature of the surface code is that errors only need to be corrected when they affect measurement outcomes**, and thus one **merely needs to identify errors**, and then correct any measurements that are affected by these errors

Digitization of Error in QEC

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi/2}\sin(\theta/2)|1\rangle$$

QEC is based upon assumption that small change in θ will be measured as bit-flip, small change in phase φ will be measured as phase flip

Quantum Circuit required **to project and an arbitrary state, $|\psi\rangle_I$ into a $\pm$ eigenstate of the Hermitian operator**, $U = U^\dagger$. The measurement result of the ancilla determines which eigenstate $|\psi\rangle_I$is projected to and since the operator, U is Hermitian and unitary can only have two eigenvalues ± 1



Analog Measurements are Projective Measurements

An error can be detected with **projective measurements**

Projective measurements **coverts analog error to digital error**

From ‘few to many’ physical qubits you can create a ‘logical qubit’

Hilbert space of logical qubit is much larger than of a physical qubit

Noise change the state of qubit and we like to detect and fix this error

We would like to protect the qubits from error, errors can be detected using stabilizer

Stabilizer are operators (usually) product of Pauli matrices

Stabilizer with eigen value of +1: No error

Stabilizer with eigen value of -1: there is an error

Projector in +1 eigen state of A $\equiv P_{+}^{A} = \frac{I+A}{2}$

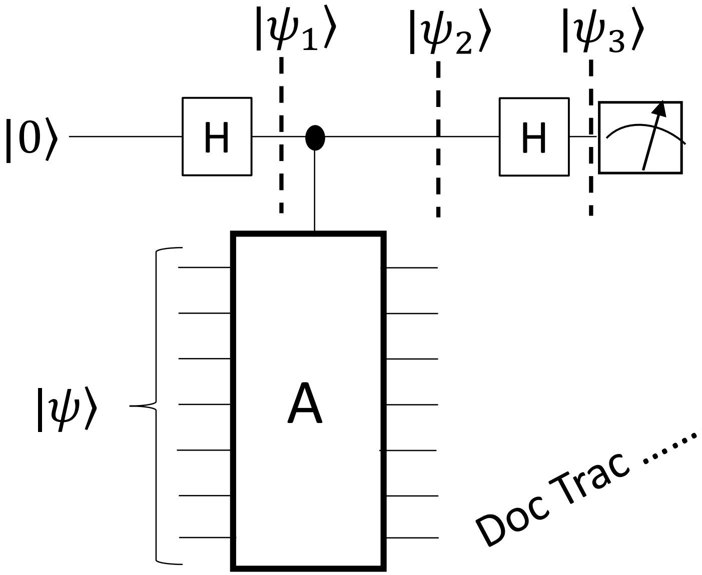
Projector in -1 eigen state of A $\equiv P_{-}^{A} = \frac{I-A}{2}$

Analog Measurements are Projective Measurements

Projector in +1 eigen state of A $\equiv P_+^A = \frac{\hat{I}+\hat{A}}{2}$ $\quad P_+^A|\psi\rangle = \frac{\hat{I}+\hat{A}}{2}|\psi\rangle$ $\quad P_+^A|\psi\rangle = \frac{1+1}{2}|\psi\rangle$

Projector in -1 eigen state of A $\equiv P_-^A = \frac{\hat{I}-\hat{A}}{2}$ $\quad P_-^A|\psi\rangle = \frac{\hat{I}-\hat{A}}{2}|\psi\rangle$ $\quad P_-^A|\psi\rangle = \frac{1-1}{2}|\psi\rangle$

IZZ and ZZI are 'three bit code' stabilizer



Doc Trac .... Error correction scheme

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$

$$|\psi_2\rangle = CA\left\{\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)\right\}$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle A|\psi\rangle)$$

$$|\psi_3\rangle = \frac{1}{2}[(|0\rangle + |1\rangle)|\psi\rangle + (|0\rangle - |1\rangle)A|\psi\rangle)]$$

$$|\psi_3\rangle = \left(\frac{|0\rangle + |1\rangle}{2}\right)|\psi\rangle + \left(\frac{|0\rangle - |1\rangle}{2}\right)A|\psi\rangle$$

$$|\psi_3\rangle = |0\rangle\left(\frac{\hat{I}+\hat{A}}{2}\right)|\psi\rangle + |1\rangle\left(\frac{\hat{I}-\hat{A}}{2}\right)|\psi\rangle$$

$$|\psi_3\rangle = |0\rangle(P_+^A)|\psi\rangle + |1\rangle(P_-^A)|\psi\rangle$$

Measured output is either +1 eigen state or -1 eigen state of projector

Repeated QEC in Surface Code

Using a range of different schemes, logical qubits can be redundantly encoded in a set of physical qubits. One such scalable approach is based on the surface code

This paper experimentally implement its smallest viable instance, capable of repeatedly detecting any single error using seven superconducting qubits, four data qubits and three ancilla qubits.

In stabilizer codes for quantum error correction, a set of commuting multiqubit operators is repeatedly measured, which projects the qubits onto a degenerate eigenspace of the stabilizers referred to as the code space. Thus, the experimental realization of quantum error detection crucially relies on high-fidelity entangling gates between the data qubits and the ancilla qubits and on the simultaneous high-fidelity single-shot readout of all ancilla qubits.

C. K. Andersen, A. Remm, S. Lazar, S. Krinner, N. Lacroix, G. J. Norris, M. Gabureac, C. Eichler, and A. Wallraff, ‘***Repeated Quantum Error Detection in a Surface Code***’, *Nat. Phys.* **16,** 875–880 (2020)

Seven Qubit Surface Code

Stabilizers of the distance-two surface code,

$$\hat{X}_{D1}\hat{X}_{D2}\hat{X}_{D3}\hat{X}_{D4}, \qquad \hat{Z}_{D1}\hat{Z}_{D3}, \qquad \hat{Z}_{D2}\hat{Z}_{D4}$$

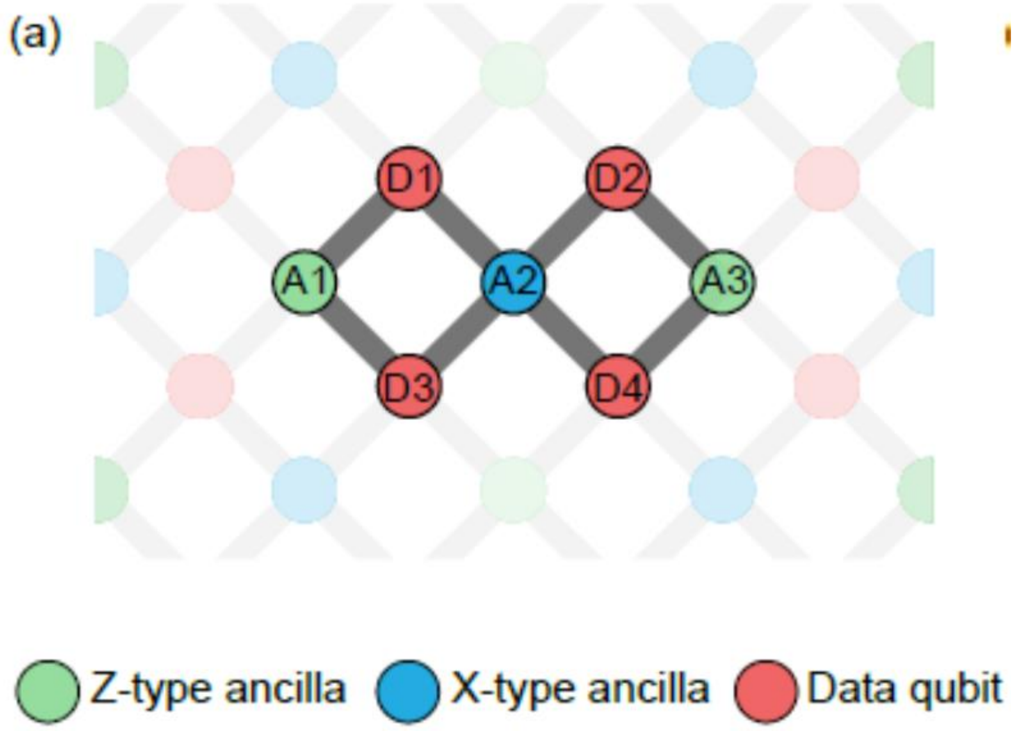
Here, we use the following logical qubit operators

$$Z_L = Z_{D1}Z_{D2}, \quad \text{or} \quad Z_L = Z_{D3}Z_{D4},$$
$$X_L = X_{D1}X_{D3}, \quad \text{or} \quad X_L = X_{D2}X_{D4},$$

such that the code space in terms of the physical qubit states is spanned by the logical qubit states

$$|0\rangle_L = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle), \tag{4}$$
$$|1\rangle_L = \frac{1}{\sqrt{2}}(|0101\rangle + |1010\rangle). \tag{5}$$

The surface code consist of a d x d grid of data qubits with d^2-1 ancilla qubit, each connected up to four data qubits. The code can detect d-1 error and correct up to [(d-1)/2] errors per cycle of stabilizer measurements.



For the code-distance d = 2, it is only possible to detect a single error per round of stabilizer measurements and once an error is detected, the error can not be unambiguously identified, e.g. one would obtain the same syndrome outcome for an X-error on D1 and on D3.

Correction of both bit- and phase-flip errors requires at least a distance-three code. In combination with faulttolerant circuits for error syndrome measurements, this guarantees that any single error on any of the constituent data and auxiliary qubits or operations can be corrected [14, 23]

Realizing Repeated Quantum Error Correction in a Distance-Three Surface Code

In an error correction cycle taking only 1.1μs, we demonstrate the preservation of four cardinal states of the logical qubit

Logical State Preparation and Measurement

Initialize the logical state

In stabilizer codes for quantum error correction [29, 30], a set of commuting multi-qubit operators is repeatedly measured, which projects the qubits onto a degenerate eigenspace of the stabilizers referred to as the code space.

Thus, the experimental realization of quantum error detection crucially relies on high-fidelity entangling gates between the data qubits and the ancilla qubits and on the simultaneous high-fidelity single-shot readout of all ancilla qubits.

Here, we utilize low-crosstalk multiplexed readout and a sequential stabilizer-measurement scheme [37] for implementing a seven qubit surface code with superconducting circuits.

In the surface code, as in any stabilizer code, errors are detected by observing changes in the stabilizer measurement outcomes. Such syndromes are typically measured by entangling the stabilizer operators with the state of ancilla qubits, which are then projectively measured to yield the stabilizer outcomes.