PHYS 512 -Quiz -01

Solution

Name: …………………………………. **ID: ………..……………………….**

Q. 1 (State Normalization) [5]

Verify each of the following state is normalized, if the state is NOT normalized, normalized the state

a) $|\psi\rangle = \frac{1}{2}|0\rangle - \left(\frac{1-i}{2}\right)|1\rangle$

Solution

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1-i}{2}\right)\left(\frac{1+i}{2}\right) = \frac{1}{4} + \frac{1-i^2}{4} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

State $|\psi\rangle$ is not a normalized state

The normalized state $|\psi'\rangle$

$$|\psi'\rangle = \left[\frac{1}{\sqrt{3/4}}\right]\left(\frac{1}{2}|0\rangle - \left(\frac{1-i}{2}\right)|1\rangle\right)$$

$$|\psi'\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{(1-i)}{\sqrt{3}}|1\rangle$$

a) $|\varphi\rangle = \frac{3i\,|0\rangle + 4|1\rangle}{5}$

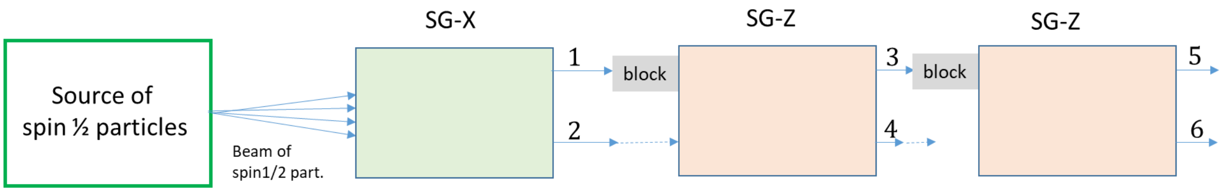
Solution

$$\left(\frac{3i}{5}\right)\left(\frac{-3i}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right) = \frac{9}{25} + \frac{16}{25} = 1$$

State $|\varphi\rangle$ is not a normalized state

Q.2 **(SG experimental Arrangement) [5]**

Following is the Stern-Gerlach experiment arrangement. For each output stage, write down the state and the probability of the measurement outcome.



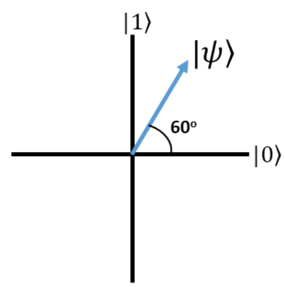
Solution

Output	State	probability
1	$\|-\rangle$	50
2	$\|+\rangle$	50
3	$\|0\rangle$	25
4	$\|1\rangle$	25
5	$\|0\rangle$	0
6	$\|1\rangle$	25

Q.3 (Change of Basis -Experimental)) [4]

A quantum state $|\psi\rangle$ is represented in the following diagram in the $|0\rangle - |1\rangle$ plane. Assuming (as is fundamental to quantum mechanics) that the state itself cannot be directly measured, only its **'components'** can be measured. Write the state $|\psi\rangle$ in a superposition of basis states $|0\rangle$ and $|1\rangle$.

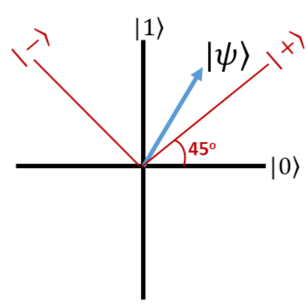
Note: no need to simplify the state expression



Solution

$|\psi\rangle = cos60|0\rangle + sin60|1\rangle$

Now if you measure the same state $|\psi\rangle$ in new basis, shown in Figure. Write the state $|\psi\rangle$ in a superposition of basis states $|+\rangle$ and $|-\rangle$.



Solution

$|\psi\rangle = cos15|0\rangle + sin1|1\rangle$

Q.4 (Orthonormality) [5]

For given two states in $|0\rangle$ and $|1\rangle$ qubit state

$$|\psi\rangle = \frac{i}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

$$|\phi\rangle = \frac{i}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

Is state $|\psi\rangle$ is orthogonal to $|\phi\rangle$?

Solution

$$\langle\phi|\psi\rangle = \left[\frac{-i}{\sqrt{2}}\langle 0| + \frac{-i}{\sqrt{2}}\langle 1|\right]\left[\frac{i}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right]$$

$$\langle\phi|\psi\rangle = \frac{1}{2}\langle 0|0\rangle + \left(-\frac{1}{2}\right)\langle 1|1\rangle = 0$$

Yes, the state $|\psi\rangle$ is orthogonal to $|\phi\rangle$

Q.5 (Entangled States) [4]

For given entangled state $|\psi\rangle$

$$|\psi\rangle = \left(\frac{1}{4} + \frac{i}{4}\right)|01\rangle + \frac{\sqrt{7}}{2\sqrt{2}}|10\rangle$$

(a) What is the probability of finding the first qubit in state $|1\rangle$?

(b) If you measure, the first qubit is state $|1\rangle$ what is/are the possible states of second qubit

Solution

(a)

$$\left(\frac{\sqrt{7}}{2\sqrt{2}}\right)\left(\frac{\sqrt{7}}{2\sqrt{2}}\right) = \frac{7}{8}$$

(b)

Second qubit for sure will be in state $|0\rangle$

Q.6 [5] Consider following state in orthonormal basis $|u_1\rangle$, $|u_2\rangle$, and $|u_3\rangle$

$$|\psi\rangle = \frac{i}{\sqrt{5}}|u_1\rangle + \frac{\sqrt{7}}{\sqrt{15}}|u_2\rangle + \frac{1}{\sqrt{3}}|u_3\rangle$$

Find the probability the state $|\psi\rangle$ collapse into state $|\varphi\rangle$, where

$$|\varphi\rangle = \frac{i}{\sqrt{2}}|u_1\rangle - \frac{i}{\sqrt{2}}|u_2\rangle$$

Solution

$$\langle\phi|\psi\rangle = \left[\frac{-i}{\sqrt{2}}\langle u_1| + \frac{i}{\sqrt{2}}\langle u_2|\right]\left[\frac{i}{\sqrt{5}}|u_1\rangle + \frac{\sqrt{7}}{\sqrt{15}}|u_2\rangle + \frac{1}{\sqrt{3}}|u_3\rangle\right]$$

$$\langle\phi|\psi\rangle = \left(\frac{-i}{\sqrt{2}}\right)\left(\frac{-i}{\sqrt{5}}\right) + \left(\frac{i}{\sqrt{2}}\right)\left(\frac{\sqrt{7}}{\sqrt{15}}\right)$$

Rest of the terms will be zero, due to orthonormality condition

$$\langle\phi|\psi\rangle = \frac{-1}{\sqrt{10}} + \frac{i\sqrt{7}}{\sqrt{30}}$$

Therefore the required probability

$$|\langle\phi|\psi\rangle|^2 = \left(\frac{-1}{\sqrt{10}} + \frac{i\sqrt{7}}{\sqrt{30}}\right)\left(\frac{-1}{\sqrt{10}} + \frac{-i\sqrt{7}}{\sqrt{30}}\right)$$

$$|\langle\phi|\psi\rangle|^2 = \frac{1}{10} + \frac{7}{30} = \frac{1}{3}$$

Q.7 [5] Consider following two state in $\mathbb{C}^3$

$$|\mathrm{a}\rangle = \begin{pmatrix} 2 \\ 4i \\ 1 \end{pmatrix}; \qquad |\mathrm{b}\rangle = \begin{pmatrix} i \\ 3 \\ 2i \end{pmatrix}$$

(i) $\langle a|b\rangle = ?$

(ii) $|a\rangle\langle b| = ?$

Solution

(i)

$$\langle a|b\rangle = \begin{pmatrix} 2 & -4i & 1 \end{pmatrix} \begin{pmatrix} i \\ 3 \\ 2i \end{pmatrix}$$

$$\langle a|b\rangle = 2i - 12i + 2i = -8i$$

(ii)

$$|a\rangle\langle b| = \begin{pmatrix} 2 \\ 4i \\ 1 \end{pmatrix} \begin{pmatrix} -i & 3 & -2i \end{pmatrix}$$

$$|a\rangle\langle b| = \begin{pmatrix} -2i & 6 & -4i \\ 4 & 12i & 8 \\ -i & 3 & -2i \end{pmatrix}$$