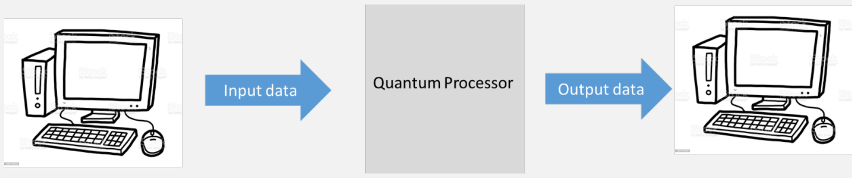
Question 1: Quantum Hardware Measurement

Circle the region on following figure where we cannot measure the data/state of hardware.

Solution:
The quantum processor (central box) is where we cannot measure the data/state directly without collapsing the quantum state. This is the region that should be circled.



Question 2: Classical to Quantum State

Write the 'quantum state' of the following 'classical state'. (Do not write the state in terms of $\alpha, \beta, \gamma, ...$, use real/complex numbers.)

$$|\text{Classical state}\rangle = \frac{1}{4}|HH\rangle + \frac{1}{4}|HT\rangle + \frac{1}{4}|TH\rangle + \frac{1}{4}|TT\rangle$$

Solution:
Converting probabilities to probability amplitudes:

$$\frac{1}{4} \text{ probability} \Rightarrow \sqrt{\frac{1}{4}} = \frac{1}{2} \text{ probability amplitude}$$

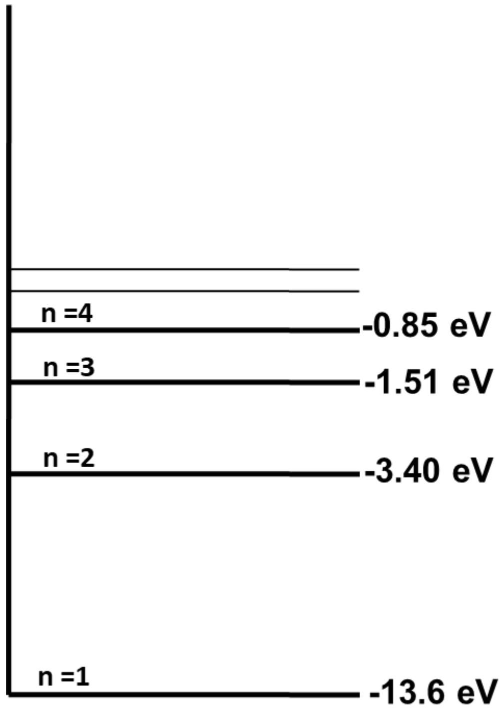
Therefore:

$$|\text{Quantum state}\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

Note: This is one possible solution that will lead to these statistics; any state varied by a global phase will lead to the same statistics.

Question 3: Energy Levels in Hydrogen Atom

Following diagram shows the energy level of electron in a hydrogen atom.



(a) Assume electron is in $n = 1$ state

1) How much energy is required to move it to $n = 2$ state?

$$\Delta E = E_f - E_i = -3.4 - (-13.6) = 10.2eV \tag{1}$$

2) How much energy is required to move it to $n = 4$ state?

$$\Delta E = E_f - E_i = -0.85 - (-13.6) = 12.75eV \tag{2}$$

(b) Energy absorption analysis

Assume electron is in $n = 1$ state, if you provide 12.2 eV of energy to this electron, where can you find the electron after it has been exposed to this 'quanta of energy'?

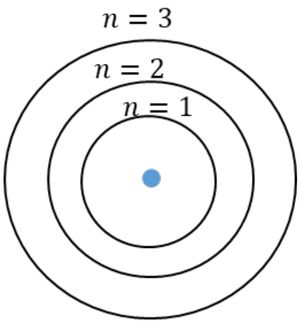
Solution:
Calculate possible transitions:

$$\Delta E_{12} = E_2 - E_1 = 10.2 \text{ eV} \tag{3}$$
$$\Delta E_{13} = E_3 - E_1 = 12.09 \text{ eV} \tag{4}$$
$$\Delta E_{14} = E_4 - E_1 = 12.75 \text{ eV} \tag{5}$$

The highest possible transition is to $n = 3$, with an excess energy of 0.11 eV that is not enough for more transitions.

Question 4: Quantum State from Probabilities

In the following Figure the electron orbits around the nucleus. Quantum mechanically the probability of finding the electron in states $n = 1$, $n = 2$, and $n = 3$ is 70%, 25% and 5% respectively. Write down the 'quantum state' of the electron around the nucleus.



Solution:
Given the state: $\alpha |1\rangle + \beta |2\rangle + \gamma |3\rangle$
From probabilities:

$$\alpha^2 = \frac{7}{10} = \frac{14}{20} \Rightarrow \alpha = \pm\sqrt{\frac{14}{20}} \tag{6}$$

$$\beta^2 = \frac{1}{4} = \frac{5}{20} \Rightarrow \beta = \pm\sqrt{\frac{5}{20}} \tag{7}$$

$$\gamma^2 = \frac{1}{20} \Rightarrow \gamma = \pm\sqrt{\frac{1}{20}} \tag{8}$$

Therefore:

$$|\psi\rangle = \sqrt{\frac{14}{20}}\,|1\rangle + \sqrt{\frac{5}{20}}\,|2\rangle + \sqrt{\frac{1}{20}}\,|3\rangle$$

Note: This is one possible solution; any state varied by a global phase will lead to the same statistics.

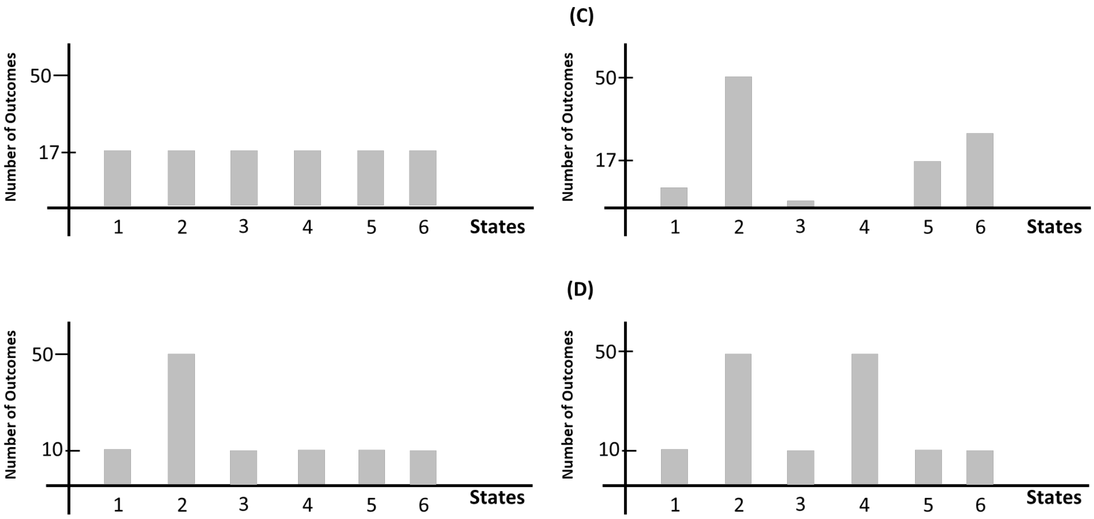
Question 5: Six-Level Quantum System

The state $|\psi\rangle$ of a six-level quantum system is:

$$|\psi\rangle = \alpha\,|1\rangle + \beta\,|2\rangle + \gamma\,|3\rangle + \delta\,|4\rangle + \kappa\,|5\rangle + \lambda\,|6\rangle$$

Where $\beta = \frac{1}{2} + \frac{i}{2}$; $\beta\beta^* = 0.5 = 50\%$

If you measure the state $|\psi\rangle$ 100 times (after preparing same state $|\psi\rangle$ again and again), which of the following is/are the possible histogram of the measurement outcomes?



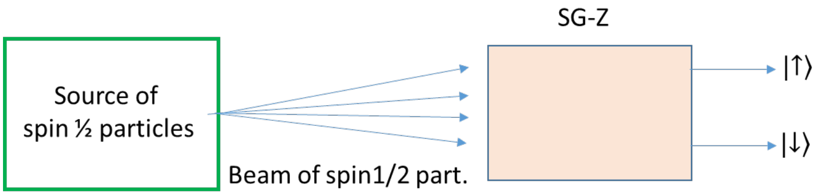
Solution:
Since $|\beta|^2 = 50\%$, state $|2\rangle$ must appear approximately 50 times out of 100 measurements.
Looking at the histograms:

- (A) Shows equal distribution (17 each) - **Not possible** $\times$
- (B) Shows state 2 with 50 counts - **Possible** ✓
- (C) Shows state 2 with 50 counts - **Possible** ✓
- (D) State 2 doesn't have 50% - **Not possible** $\times$

Answer: Histograms (B) and (C) are possible

Question 6: Stern-Gerlach Experiments

(a) Quantum state from SG-Z measurements



If source produces 1000 spin-1/2 particles and after passing through SG-Z, 500 particles measured with $|\uparrow\rangle$ and 500 measured with $|\downarrow\rangle$. Write down the 'quantum state, $|\psi\rangle$' of the spin-1/2 particles from the source.

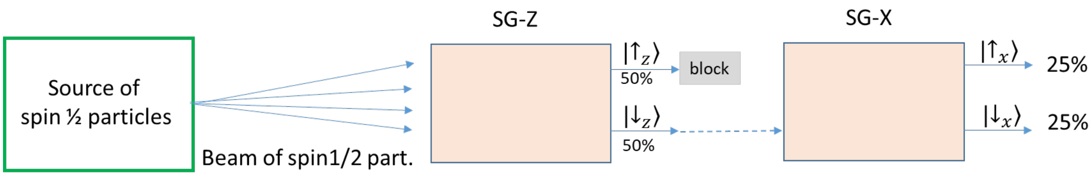
Solution:
Equal probabilities mean equal amplitudes:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$

Note: This is one possible solution; any state varied by a global phase will lead to the same statistics.

(b) Sequential SG apparatus

Probabilities after each SG apparatus are given in the following diagram. Write down the quantum state $|\downarrow_z\rangle$ in terms of $|\downarrow_x\rangle$ and $|\uparrow_x\rangle$.



Solution:
Since the $|\downarrow_z\rangle$ state splits equally into $|\uparrow_x\rangle$ and $|\downarrow_x\rangle$ states:

$$|\downarrow_z\rangle = \frac{1}{\sqrt{2}}|\uparrow_x\rangle + \frac{1}{\sqrt{2}}|\downarrow_x\rangle$$

With only 50% of the original count passing through (since $|\uparrow_z\rangle$ is blocked).
Note: This is one possible solution; any state varied by a global phase will lead to the same statistics.