

## Assignment – 01 - PHYS 514

Due Monday 19 January – 12:00 mid night

*(Late submission will not be accepted)*

**Submit Question # 3, 4, 6, and 7 ONLY**

Please submit your solution via BB or email. Ensure that your **full name and student ID** are included on your submission

### **Q.1 Plasma Frequency**

Following expression is to find to find the ‘plasma frequency -  $\omega_p$ ’

$$\omega_p^2 = \frac{ne^2}{m\epsilon_0}$$

Where ‘m’ is the effective mass of electron (you use rest mass of electron) and ‘n’ electron density.

- Find the plasma frequency in a metal (of your choice)
- Find the temperature corresponding to your calculated value of  $\omega_p$ .

(Remember  $E = hf = h\frac{2\pi}{\omega}$  and  $E = K_B T$ )

### **Q.2**

Briefly explain the concept of ‘skin depth’, in reference to electromagnetic radiations

### **Q.3 Commutation Relation**

Using the commutation relation between  $\hat{x}$  and  $\hat{p}$  show that

$$[a, a^\dagger] = 1$$

### **Q.4 Eigen State of Oscillator Hamiltonian**

If

$$\hat{H}|n\rangle = E_n|n\rangle$$

Where  $|n\rangle$  are the Fock states of LC oscillator, using commutation relation between  $a$  and  $a^\dagger$  show that

$$\hat{H}\hat{a}|n\rangle = (E_n - \hbar\omega)\hat{a}|n\rangle$$

### **Q.5 Hamiltonian and ladder operator**

Using commutation relation of the ladder operator, show that

$$\hat{H} = \frac{\hbar\Omega}{2} [a^\dagger a + a a^\dagger]$$

can be expressed as

$$\hat{H} = \hbar\Omega [a^\dagger a + 1/2]$$

### **Q.6 Ladder Operator**

(a) Show that

$$[\hat{n}, \hat{a}^\dagger] = \hat{a}^\dagger$$

$$[\hat{n}, \hat{a}] = -\hat{a}$$

(b) Find the normalization constant  $C_1$  and  $C_2$

$$\hat{a}|n\rangle = C_1|n-1\rangle$$

$$\hat{a}^\dagger|n\rangle = C_2|n+1\rangle$$

### **Q.7 Number Operator**

Consider a cavity that contains superposition of two Fock states described by

$$|\psi\rangle = \sqrt{0.99}|0\rangle + \sqrt{0.01}|100\rangle$$

- What is the average number of photons in the cavity?
- If you annihilate a photon by acting annihilation operator  $\hat{a}$  on this state, then how many photons remain in the cavity? Interpret the result.