Assignment – 02 - PHYS 514

Due Thursday 29 January – 12:00 mid night

(Late submission will not be accepted)

Submit solutions of problem # 1, 4, and 5 ONLY

Please submit your solution via BB or email. Ensure that your **full name and student ID** are included on your submission

Q.1 [Lagrange and Euler-Lagrange Equation]

(a) Write down the Lagrange equation of LC circuit in terms of $q \; and \; \dot{q}$.

(b) Using following Euler-Lagrange equation

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

Show that

$$\ddot{q} = -\Omega^2 q$$

Where $\Omega = \frac{1}{\sqrt{LC}}$

(c) Find conjugate momentum for (position) coordinate q (using Lagrange of part a)

(d) Find the Hamiltonian of the LC circuits.

Q.2 [Hamiltonian of LC Oscillator]

(a) Show that the

$$\hat{H} = \frac{1}{2C}\hat{Q}^2 + \frac{1}{2L}\hat{\varphi}^2$$

can be expressed as

$$\hat{H} = \frac{\hbar\Omega}{2}[a^\dagger a + aa^\dagger]$$

Where $a = i\frac{1}{\sqrt{2C\hbar\Omega}}\hat{Q} + \frac{1}{\sqrt{2L\hbar\Omega}}\hat{\varphi}$ and $a^\dagger = -i\frac{1}{\sqrt{2C\hbar\Omega}}\hat{Q} + \frac{1}{\sqrt{2L\hbar\Omega}}\hat{\varphi}$

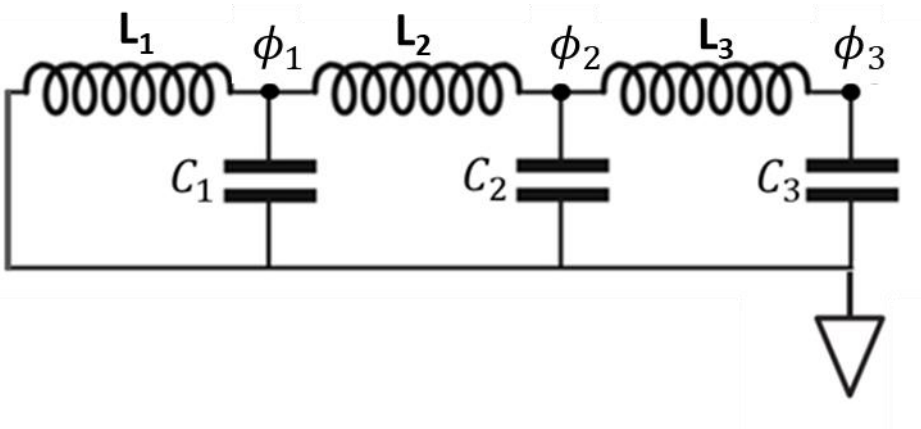
Q.3 [Expectation values of Q and ϕ]

Using 'ladder operation' expression for Q, find expectation value for Q $\left(\langle\hat{Q}\rangle\right)$ and Q^2 $\left(\langle\hat{Q}^2\rangle\right)$ for vacuum (zero photon) state.

Q.4 [Zero Point Fluctuation (ZPF)]

Calculate the zero-point fluctuation in charge, Q_{ZPF} for an LC oscillator circuit. Calculate the Φ_{ZPF} as well. Assume L= 50 nH and C = 70 pF. Express Q_{ZPF} in terms of number of electrons (i.e. Q_{ZPF}/e).

Q. 5 [Lagrange and Hamiltonian of Quantum Circuit]



(a) Write down the Lagrange of the above circuit in-terms of nodal flux ϕ
(b) Find the momentum conjugate, Q, to the flux ϕ $(1, 2, \text{and } 3)$
(c) Write down the commutation relation between Q and ϕ
(d) Find the Hamiltonian of the above circuit
(Hint: $H = \sum_{1=1}^{3} Q_i \dot{\phi}_i - L$
(e) Write down the Hamiltion's equations of motion