Assignment – 03 - PHYS 514

Due Thursday Feb 05 – 12:00 mid night

(Late submission will not be accepted)

Submit Question # 2, 3, and 4 ONLY

You can submit the hard copy or soft-copy via email – please make to write your name and ID

Q.1 Commutation Relation

Using ladder expression for both $\hat{n}$ **and** $\hat{E}_x$ Show that

$$\left[\hat{n}, \hat{E}_x\right] = \epsilon_0 \sin(kz)\ (\hat{a}^\dagger - \hat{a})$$

Q.2 Matrix Representation

a) Using basic definition of ladder operators express $\hat{a}\ and\ \hat{\hat{a}}^\dagger$ in matrix form

b) Use the matrix form of $\hat{a}\ and\ \hat{\hat{a}}^\dagger$ (from part (a)
Verify the following (1- 6 only), where $\sigma_x, \sigma_y, and\ \sigma_z$ are usual Pauli operators

1. $\hat{\hat{a}}^\dagger - \hat{a} = -i\sigma_y$
2. $\hat{\hat{a}}^\dagger + \hat{a} = \sigma_x$
3. $\left(\hat{\hat{a}}^\dagger - \hat{a}\right)^2 = -\sigma_z$
4. $\left(\hat{\hat{a}}^\dagger + \hat{a}\right)^2 = -\sigma_z$
5. $\left(\hat{\hat{a}}^\dagger + \hat{a}\right)^3 = 3\sigma_x$
6. $\left(\hat{\hat{a}}^\dagger + \hat{a}\right)^4 = -6\sigma_z$
7. $\left(\hat{a}_i^\dagger - \hat{a}_i\right)\left(\hat{a}_j^\dagger - \hat{a}_j\right) = -\sigma_{iy}\sigma_{jy}$
8. $\left(\hat{a}_i^\dagger + \hat{a}_i\right)\left(\hat{a}_j^\dagger + \hat{a}_j\right) = -\sigma_{ix}\sigma_{jx}$
9. $\left(\hat{a}_i^\dagger - \hat{a}_i\right)^3\left(\hat{a}_j^\dagger + \hat{a}_j\right) = 3\sigma_{ix}\sigma_{jx}$
10. $\left(\hat{a}_i^\dagger + \hat{a}_i\right)^2\left(\hat{a}_j^\dagger + \hat{a}_j\right)^2 = \sigma_{iz}\sigma_{jz} - 2\sigma_{iz} - 2\sigma_{jz}$

Q.3 Problem 2.5 (Book – Intro to Quantum Optics, pages 40-41)

Q.4 Problem 2.7 (Book – Intro to Quantum Optics, pages 40-41)

Q.5 Problem 2.8 (Book – Intro to Quantum Optics, pages 40-41)