

PHSY 514
Final Exam – Part – I (take home)

Q.1

The ‘dressed states’ eigenstates associated cavity-qubit system is given by

$$|n, -\rangle = \cos(\theta_n) |g\rangle|n+1\rangle - \sin(\theta_n) |e\rangle|n\rangle$$

$$|n, +\rangle = \sin(\theta_n) |g\rangle|n+1\rangle + \cos(\theta_n) |e\rangle|n\rangle$$

- (a) What are the eigen states when qubit frequency ω_q is equal to cavity frequency ω_c
- (b) What is the energy of the eigen states when $\omega_q = \omega_c$
- (c) Briefly discuss the condition in which $\theta_n = 0$. Rewrite the dressed states for $\theta_n = 0$ and briefly explain how you can create this condition.

Attempt Q.2 or Q.3

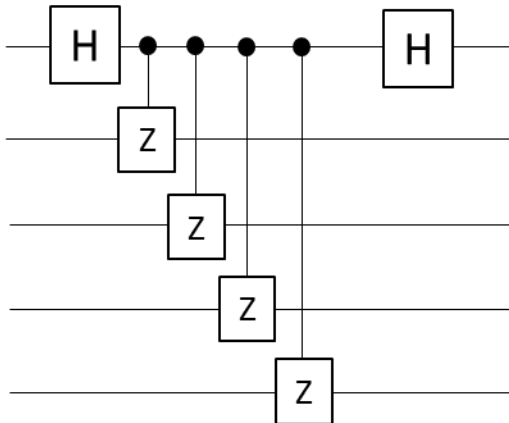
Q.2

- (a) What do you mean by the quality factor of a cavity and CPW. Briefly explain how do you measure the quality factor of a cavity/CPW
- (b) What is the difference between Rabi measurements and Ramsi measurements
- (c) Briefly explain the T1 and T2 measurements of a qubit

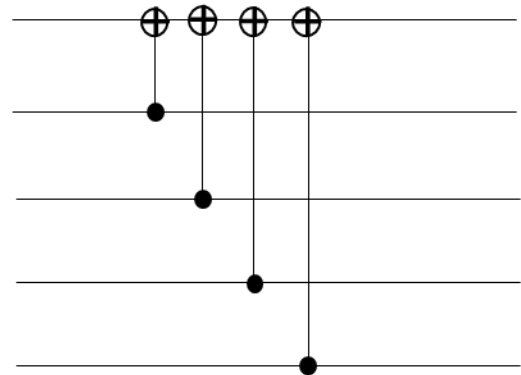
Q.3

Demonstrate, either mathematically or through logical (gate) operation, that circuits A and B are equivalent. (For logical operation use ONLY combination of the input states of your choice).

A)



B)



Q. 4

Consider two operators ($\hat{H}_{JC}$ is Jaynes Cummins Hamiltonian)

$$\hat{H}_{JC} = \hbar\omega_r \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_q}{2} \hat{\sigma}_z + i\hbar g(\hat{a}\hat{\sigma}^\dagger - \hat{a}^\dagger \hat{\sigma})$$

$$\hat{E} = \hat{a}^\dagger \hat{a} + \hat{\sigma}^\dagger \hat{\sigma}$$

(a) Show that

$$[\hat{H}_{JC}, \hat{E}] = 0$$

(b) Do $\hat{H}_{JC}$ and $\hat{E}$ share common eigen states? (Y/N)

(c) Find that following states are eigen states of $\hat{E}$ and find corresponding eigen values.

$$\hat{E}|0, g\rangle = ?$$

$$\hat{E}|n, g\rangle = ?$$

$$\hat{E}|n-1, e\rangle = ?$$

(d) Find

$$\hat{H}_{JC}|0, g\rangle = ?$$

$$\hat{H}_{JC}|n, g\rangle = ?$$

$$\hat{H}_{JC}|n-1, e\rangle = ?$$

(e) In part (d) n is the Fock state and ‘g and e’ are ground and excited states of a qubit; write five possible states of cavity(resonator)-qubit coupled system.

Q.5

Consider the following Hamiltonian (JJ-based non-linear osc.)

$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos \hat{\phi} \quad (1)$$

(a) By substituting $\hat{n} = \frac{\hat{Q}}{2e}$ and $E_c = \frac{1}{2} \frac{(2e)^2}{C} = \frac{2e^2}{C}$ show that above $\hat{H}$ can be expressed as (remember, here $\hat{n}$ is not a number operator)

$$\hat{H} = 4E_c \hat{n}^2 - E_J \cos \hat{\phi} \quad (2)$$

- (b) Express Hamiltonian of equation (2) in matrix form in 'n basis' where n is number of cooper pairs. $\hat{n}$ is number of cooper-pair operator and satisfy the following

$$\hat{n}|n\rangle = n|n\rangle$$

$$e^{i\hat{\phi}}|n\rangle = |n+1\rangle$$

$$e^{-i\hat{\phi}}|n\rangle = |n-1\rangle$$

Hint: Matrix form of Hamiltonian (where m, n goes from $0 \rightarrow \infty$)

$$\hat{H} \Rightarrow H_{mn} = \langle m|\hat{H}|n\rangle$$

Express $\cos\hat{\phi} = \frac{e^{i\hat{\phi}} + e^{-i\hat{\phi}}}{2}$

Find the matrix element (find few at least 6 non-zero entries) and complete the matrix form of the required Hamiltonian.

- (c) Rewrite the Hamiltonian matrix when $n = 0$ and 1 only

Q.6

Answer the following questions (no need to explain and drive anything, unless asked for)

- Write down Hamiltonian of a superconducting qubit,
- Write down Hamiltonian of a transmon qubit.
- Write down Jaynes Cumming (JC) Hamiltonian under rotating wave approximation,
- Write down the energy eigen states and energy eigen values of JC-Hamiltonian under RWA approximation.
- Write down the Hamiltonian of the coupled system (cavity/CPW resonator-qubit) under dispersive regime.
- Will you observe Rabi oscillations in both JC and dispersive regime Hamiltonian of coupled system (Y/N).
- Write down the Hamiltonian of LC (linear) circuit in ladder operator (no need to drive)
- Express the above Hamiltonian (in part g) in matrix form using number basis (Fock states).
- Briefly explain how dispersive regime coupling can be used to read the state of a qubit (you can support answer with equation as well)
- Briefly explain how dispersive regime coupling can be used to measure the number of photons in cavity/CPW resonator. (you can support answer with equation as well).
- Is there any effect of detuning (Δ) on the Rabi frequency? (Y/N)
- In the Rabi frequency depends upon the number of excitations in cavity (i.e. Fock state)? (Y/N)
- Experimentally, how would you verify the non-linearity of the qubit? Using (tick all possible)

I. single-tone spectroscopy,

- II. two-tone spectroscopy,
- III. combination of both (I and II)?

(n) What do you mean quantum gate fidelity and Quantum Process Tomography?