

PHYS 514
Practice Question Set-2

Q.1 Density Matrix

For a quantum state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

Show that the density matrix ρ

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2}(I + \vec{a} \cdot \vec{\sigma}) = \begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix}$$

Express the above ρ matrix in terms of α and β

Q.2 Gate Operation and Rotation

(a) Write down the output of the following

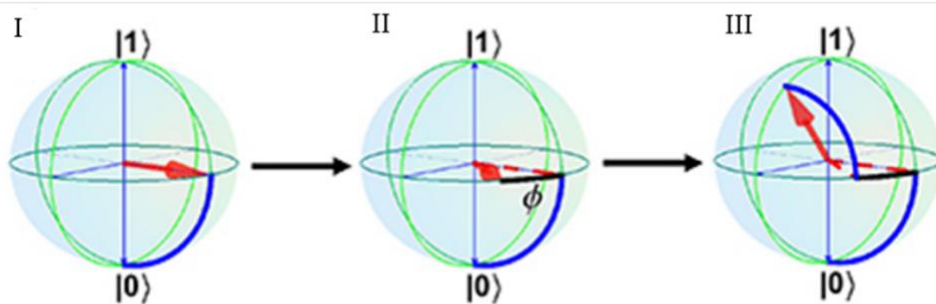
$$\text{HZX} |1\rangle =$$

(b) Write down the ‘pulses’ required to execute above said operation (of HZX) in correct order.

(c) Write down single rotation (if possible) that can execute all three gate operations HZX

(d) ‘Pure phase’ of above quantum state (part (b)) increased by $\pi/4$ before you measure the quantum state. How you will measure this quantum state (what pulse you will apply) and what is measurement outcome.

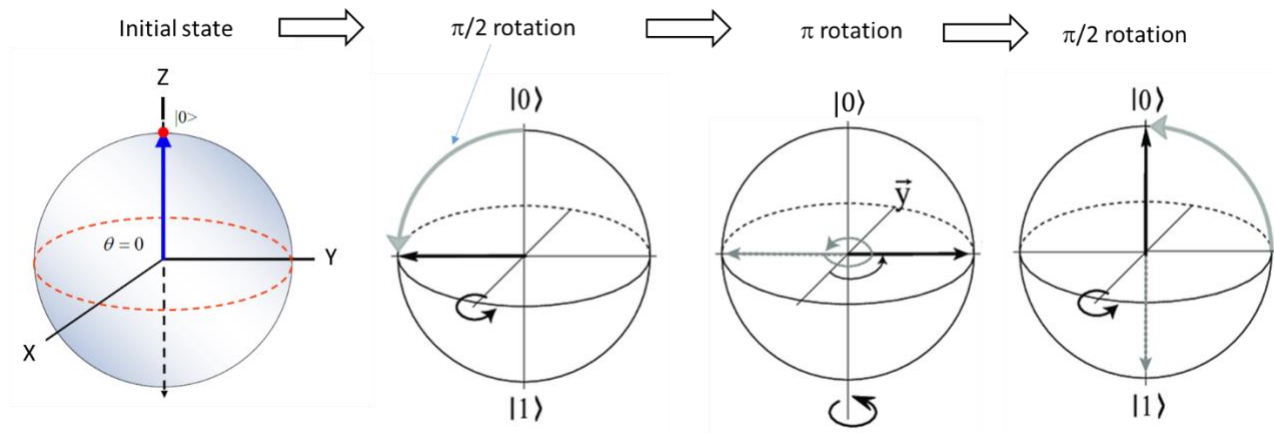
Q.3 Rotation and Measurement of a Quantum State



(a) What pulse you need to apply to go from Fig I (state $|0\rangle$) to Fig II (state $|+\rangle$)

(b) Before the measurements (Fig II to III) the phase of the state changes by $\phi = 30^\circ$ (as shown in Fig). What measurement pulse you will apply and what is the measurement outcome.

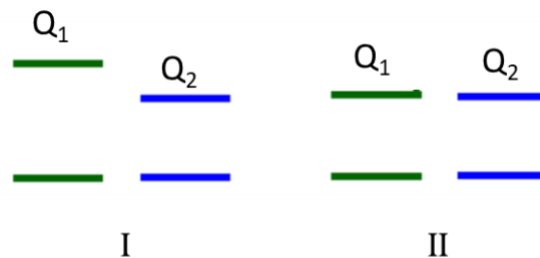
Q.4 Rotation and Measurement of a Quantum State



Write down the pulses required (in correct sequence) to create the required state in each step.

Q.5 Qubits Entanglement

(a) Two qubits energy diagrams shown below are 'coupled' (through capacitor or resonator)



In which of the cases (I or II) the qubits are entangled?

(b) Two qubits that are places far away from each other and NOT connected to each other, direct or indirect



Above two qubits are entangled (Y/N)

Q.6 Qubits Entanglement

Consider the following Hamiltonian for a ‘coupled qubit and resonator’

$$H = \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\omega_q + \hbar g(a\sigma^+ + a^\dagger\sigma^-) \dots\dots\dots(1)$$

Where ω_r and ω_q are resonant frequencies of the resonator and the qubit

- (a) In equation (1) specify the interaction Hamiltonian
- (b) What is $\Delta (= |\omega_q - \omega_r|)$ in terms of g in
 - (i) non-dispersive regime
 - (ii) dispersive regime
- (c) Rewrite the equation (1) for a direct qubit-qubit coupling

Q. 7 Rotation Operation

Apply the following pulses to a qubit in state $|0\rangle$ and write down the new state of the qubit

- (a) $Y(-\pi/2)$
- (b) $X(\pi)$
- (c) $X(\pi/2)$
- (d) $Y(-\pi/2)X(-\pi/2)Y(\pi/2)$
- (e) $X(\pi) Y(\pi/4)$

Q.8 Rotation Operation

Consider a qubit initially in state $|0\rangle$,

- (a) Specify the pulse, with pulse duration and rotation axis, that you will use to convert the initial state to $|1\rangle$
- (b) Specify a ‘suitable pulse’ that can change the state $|1\rangle$ to $|+\rangle$
- (c) Now the phase of $|+\rangle$ state changes with time. Explain how you can measure the change in phase of the $|+\rangle$ state

Q.9 Measure the State of a Qubit

A qubit is prepared in state

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

Approximately locate this state on Bloch sphere.

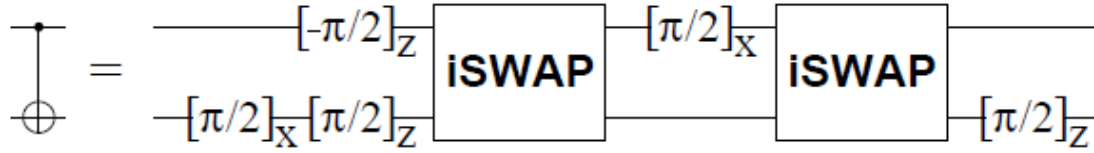
Explain how you will experimentally *verify* the prepared state, write down the steps you will follow.

Q.10 Rotation and Gate Operation

SWAP gate ‘is a bit flip’ gate in two-qubit system. Briefly explain how you will use two entanglement qubits to perform this operation.(You may write down the steps you will follow).

Q.11 Two Qubit Gate Implementation

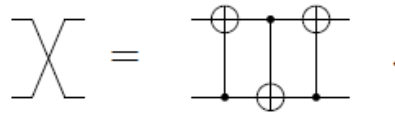
CNOT gate can be implemented using different rotations and iSWAP gates shown in Fig. For input $|1\rangle$ (top) $|0\rangle$ (bottom), using matrices (given in additional information) for each operation, show that the circuit on RHS is equivalent to CNOT gate.



Where [Rotation]_{axis of rotation}

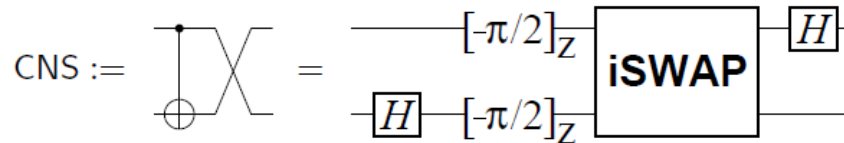
Q.12 SWAP and CNOT Gate

Show that SWAP gate can be implemented using multiple CNOT gates



Q. 13 CNOT + SWAP Operation

CNOT+SWAP ($\equiv$ CNS) gate can be implemented using different rotations and iSWAP gates shown in Fig. For input $|1\rangle$ (top) $|1\rangle$ (bottom), verify the validity of the circuit



Additional Information

Rotation along Z-axis $Z(\theta) = e^{iz\theta/2} = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$

Rotation along X-axis $X(\theta) = e^{ix\theta/2} = \begin{pmatrix} \cos(\theta/2) & -i \sin(\theta/2) \\ -i \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$

Rotation along Y-axis $Y(\theta) = e^{iy\theta/2} = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$

iSWAP $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$