

PHYS 514

Practice Questions Set - 3

Q.1 Rotation of a State

Consider the following rotations along X, Y and Z-axis

$$\text{Rotation along Z-axis} \quad Z(\theta) = e^{iz\theta/2} = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$\text{Rotation along X-axis} \quad X(\theta) = e^{ix\theta/2} = \begin{pmatrix} \cos(\theta/2) & -i \sin(\theta/2) \\ -i \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

$$\text{Rotation along Y-axis} \quad Y(\theta) = e^{iy\theta/2} = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

Show that rotation of a state along z-axis can be replaced by combination of X and Y rotations i.e.

$$Y\left(-\frac{\pi}{2}\right)X(\theta)Y\left(\frac{\pi}{2}\right) = Z(\theta)$$

Q.2 Cavity-Qubit Coupling

Consider the following Jaynes-Cumming Hamiltonian

$$H = \hbar g(a\sigma_+ + a^\dagger\sigma_-)$$

Where $a, a^\dagger$ are lowering and raising operators, respectively (for the resonator) and for qubit

$$\sigma_+|0\rangle = |1\rangle$$

$$\sigma_-|1\rangle = |0\rangle$$

$$\sigma_+|1\rangle = \sigma_-|0\rangle = 0$$

(a) Show that

$$H|n, 0\rangle = \hbar g\sqrt{n}|n-1, 1\rangle$$

$$H|n, 1\rangle = \hbar g\sqrt{n+1}|n+1, 0\rangle$$

Remember $|n, 0\rangle \equiv |n\rangle \otimes |0\rangle$

(b) Now consider the case where cavity starts in vacuum state $|0\rangle$ and the qubit is initially in the excited state $|1\rangle$, using results of part (a) show that:

$$H|0, 1\rangle = \hbar g|1, 0\rangle$$

$$H^2|0, 1\rangle = (\hbar g)^2|0, 1\rangle$$

(c) Interaction Hamiltonian (Jaynes-Cumming) is time-independent, therefore unitary evolution operator can be expressed as

$$U(t) = \exp\left(-\frac{it}{\hbar}H\right)$$

Using Jaynes-Cumming Hamiltonian show that

$$U(t) = \exp\left(-\frac{it}{\hbar} H\right) = \sum_{j=0}^{\infty} \left(-\frac{it}{\hbar}\right)^j \frac{H^j}{j!}$$

(d) For a given initial state $|\psi(0)\rangle = |0, 1\rangle$, show that the state of the system after time t will be;

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = \cos(gt)|0, 1\rangle - i \sin(gt)|1, 0\rangle$$

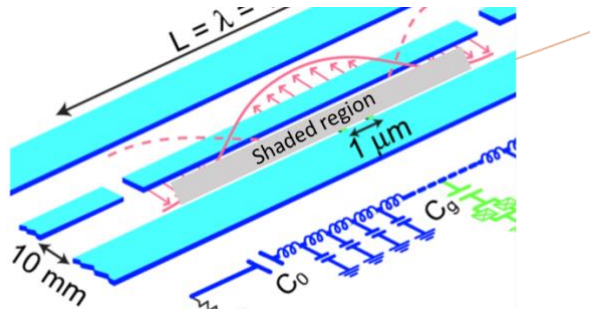
(e) Part (d) shows that the excitation, initially in the qubit subsystem, periodically oscillates back and forth from the qubit to the cavity (resonator) mode. Calculate how much time is required for the excitation to transfer *completely* from qubit to the cavity (resonator)

(f) Calculate the output state at time $t = \frac{\pi}{4g}$. Explain why this is a special type of state.

Q.3 Zero Point Fluctuation (ZPF)

Calculate the zero point fluctuation in Voltage V_{ZPF} for an LC oscillator circuit where $L = 50$ nH and $C = 70$ pF.

Further, Assume the LC oscillator (the resonator) in following Fig is in vacuum (zero photon) state. Find the average electric field, E in the shaded region



Q.4 Coherent State

A coherent state, $|\alpha\rangle$, is the eigenstate of operator $\hat{a}$:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

(g) coherent state, $|\alpha\rangle$ is the eigenstate of $\hat{a}^\dagger$ as well (Y/N)

(h) $\hat{a}$ is a lowering operator, briefly explain how the coherent state $|\alpha\rangle$ can be an eigenstate of operator $\hat{a}$

Q.5 Time Evolution of SHO

We know how quantum state evolve with time, i.e.

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = \exp\left[\frac{-i\hat{H}t}{\hbar}\right]|\psi(0)\rangle$$

And if

$$|\psi(0)\rangle = |n\rangle$$

Then (you did it in last HW)

$$|\psi(t)\rangle = \exp[-in\omega t] |n\rangle$$

From above equation it is clear that ‘the **quantum phase** of the eigenstate, $|n\rangle$ ’ evolves at rate which is proportional to n .

- (a) In above expression we assumed $\hat{H} = \hbar\omega_0\hat{a}^\dagger\hat{a}$ *instead of* $\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$, (same you used in last HW). If you use original $\hat{H}$ (with $+\frac{1}{2}$), is there any effect on ‘**quantum phase evolution**’ with time (of $|\psi(t)\rangle$). Explain it briefly.

- (b) If we prepare following initial state

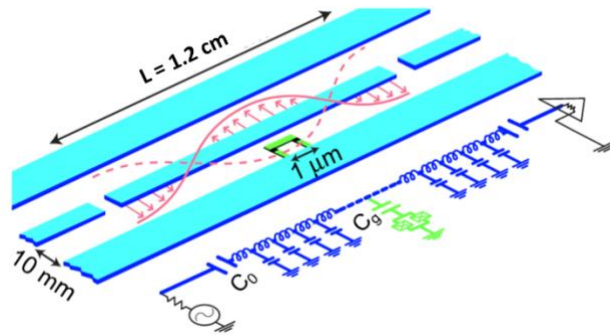
$$|\psi(0)\rangle = \sum_n c_n |n\rangle$$

Then each amplitude in the superposition, c_n , evolve at same rate or at different rate, explain.

Q.6 Resonator

Consider a coplanar waveguide cavity formed between two coupling capacitors, a central conducting strip (shown in Fig below).

- (a) If length of central strip is 1.2 cm, find the frequency of *given harmonic* and first harmonic.
(b) Find the energy of first harmonic



Q.7 Superconductivity

Briefly describe the following in reference to superconductors

- (i) Zero resistance
- (ii) Two-fluid behavior
- (iii) Energy gap
- (iv) Meissner effect
- (v) Long Range Coherence

Q.8 Josephson Energy

- (a) Using two Josephson equations derive the following expression for the Josephson energy

$$E = -E_J \cos\left(\frac{2\pi\Phi}{\Phi_0}\right) = -E_J \cos(\Delta\varphi)$$

- (b) Write down the Hamiltonian expression for the Josephson junction; clearly specify the conjugate variable and the commutation relation between them.
- (c) For particular Josephson junction if $I_c = 2 \mu A$. Find the phase difference $\Delta\varphi$, across the superconducting pads when current across the junction $I = 1.2 \mu A$
- (d) Under above given condition (in part (c)) find the value of Josephson inductance L_J

Q.9 Transmon Qubit

Following is energy spectrum of the ‘cooper pair box’ as a function of offset charge. Which of the graph is of transmon qubit. Briefly explain why ‘cooper pair box’ in transmon regime is the best choice for a qubit.

